Social Network Analysis

Paulo Serôdio

University of Essex

Content

- Introduction to Core Social Network Concepts
  - Overview of the field and the tools
  - Mathematical foundations
  - SNA Data & Survey Design
  - Centrality
  - Social Capital
  - Cohesion
  - Subgroups
  - Equivalence (Role & Position)
  - Hypotheses testing
- Introduction to network analysis in UCINET

History

Previous instructors: Steve Borgatti (Kentucky) & Rich DeJordy (Fresno State)

Objectives

- Build intuition
- Expose key concepts
- Highlight big questions
- provide abstract examples
- Some pointers to other studies
- *NOT* a substitute for technical work

Introduction

- Name
- Affiliation
- Discipline
- SNA Experience/Knowledge
- Phenomena of interest

What Defines SNA?

- Phenomenon studied
  - distinctive type of data
- Perspective taken
  - Perhaps one perspective, but multiple theories
- Methodological toolkit
  - new concepts, new tools

Reasoning about Networks

- What can achieve from studying networks?
  - Patterns and statistical properties of network data;
  - Design principles and models;
  - Understand the organisation of networks;
- How can we reason about networks?
  - **Empirical** : study data; measure and quantify;
  - **Mathematical** Models: graph theory & stats, distinguish surprising from expected phenomena
  - **Algorithms** : for hard computational challenges

how mathematicians reason about networks

- Mathematicians are concerned with the abstract structure of a graph
- Mathematicians define operations to analyze and manipulate graphs. Moreover, they develop theorems based upon structural axioms.

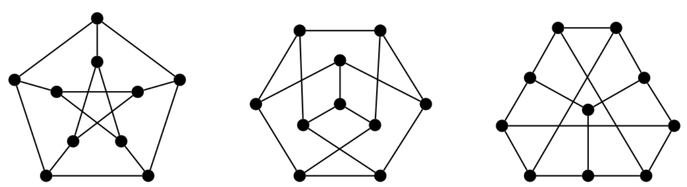
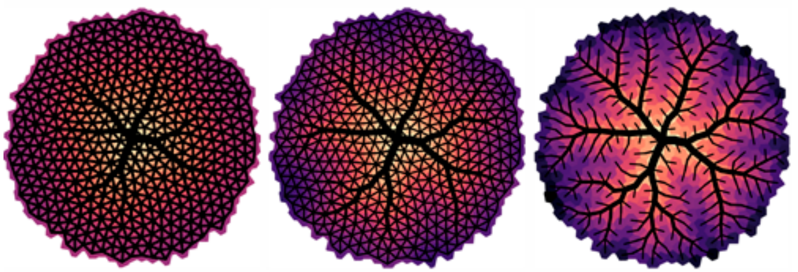


Figure 0.7: Three isomorphic drawings of the infamous Petersen graph!

how physicists reason about networks

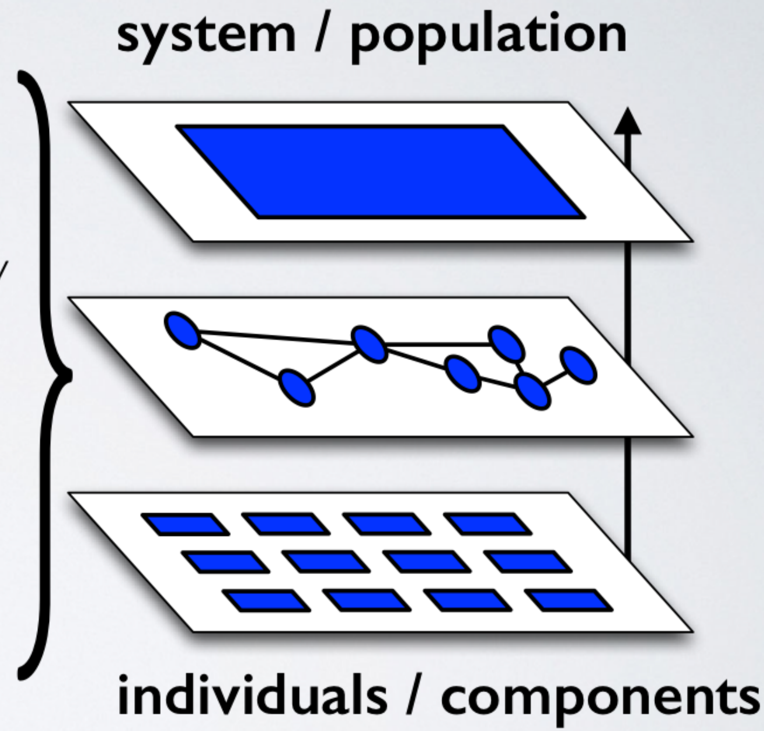
- Physicists are concerned with modeling real-world structures with networks.

- Physicists define algorithms that compress the information in a network to more simple values (e.g. statistical analysis).



what are networks?

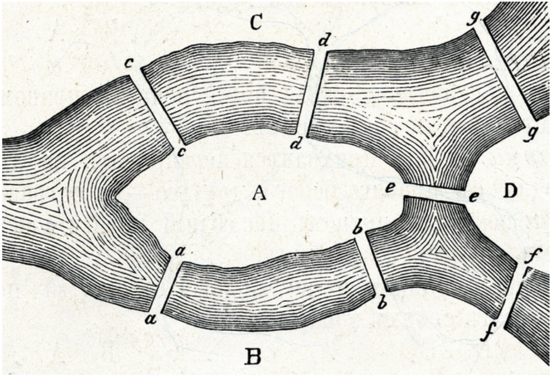
- an approach
- a mathematical representation
- provide structure to complexity
- *structure above*
  individuals / components
- *structure below*
  system / population



Graph Theory Beginnings: Leonard Euler

- Swiss mathematician and logician (1707 - 1783)
- Network analysis begins with solution to the “Bridges of Königsberg” question in 1735

The Seven Bridges of Königsberg



Big Question: Can one walk across all seven bridges and never cross the same one twice?

Definition: an Euler path walks through a graph without revisiting edges; an Euler circuit is an Euler path that starts and stops at the same vertex.

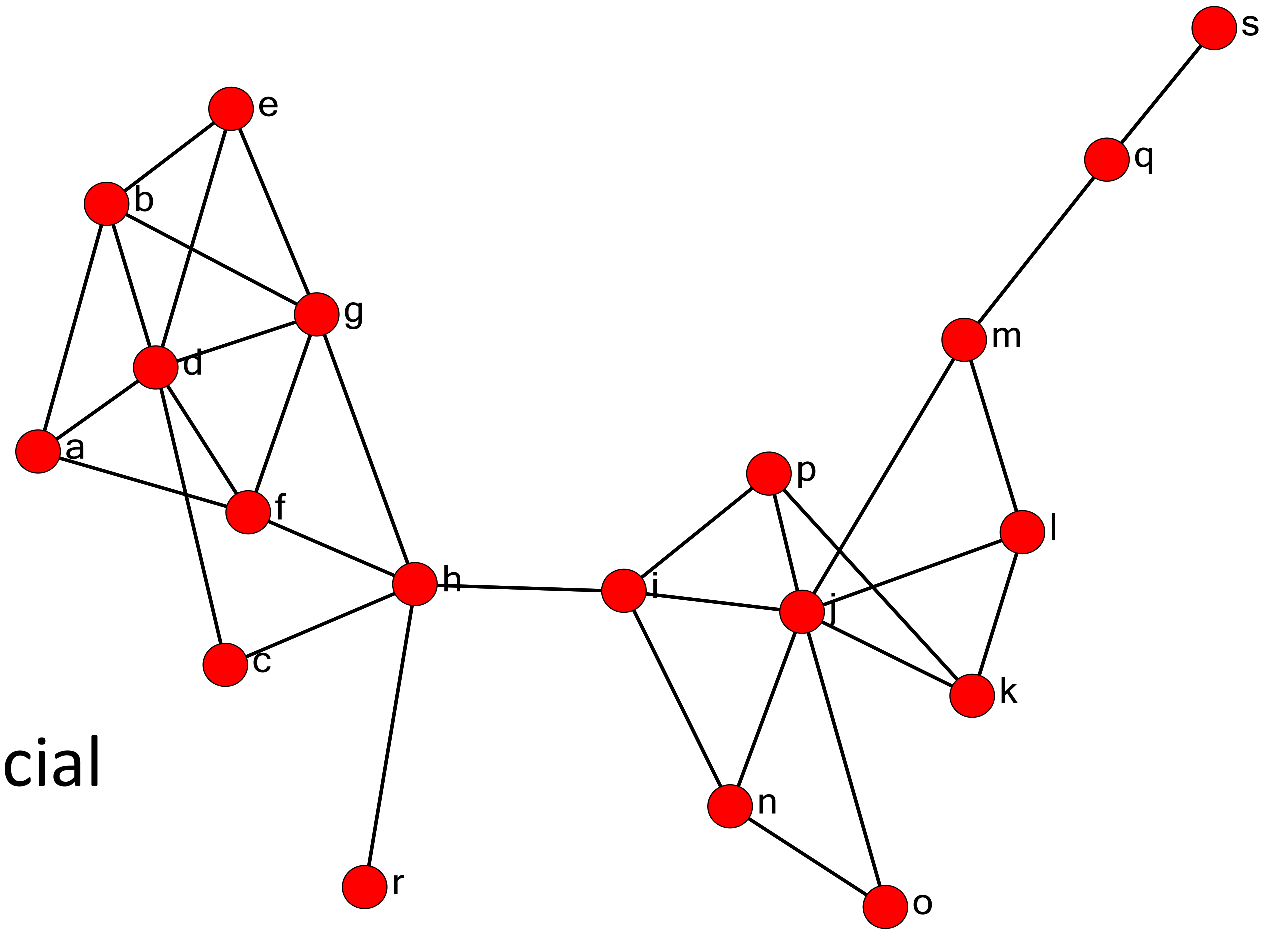
History of SNA

- 1736- Euler
- 1930s- Sociometry
- 1940s Psychologists
- 1950s & 60s Anthropologists
- 1970s Rise of Sociologists
  - Small Worlds, Strength of weak ties
- 1980s IBM computation
  - Computer programs developed
- 1990s Ideas spread
  - UCINET released, spread of network analyis to multiple fields, social capital, embedded ties
- 2000s Physicists jump on the bandwagon

What is a network?

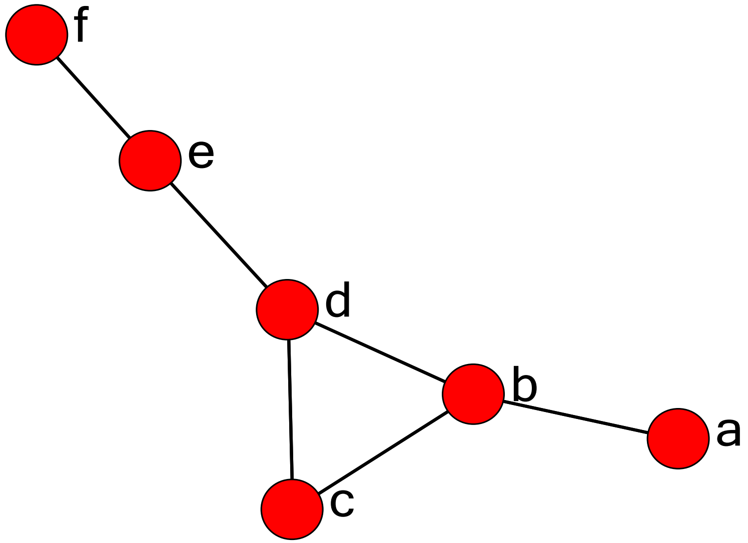
Network

- Set of nodes
- Set of ties among them
- Ties interlink through common nodes
  - Resulting in paths
- In social network analysis, ties typically represent a social relation
  - E.g., kinship, family



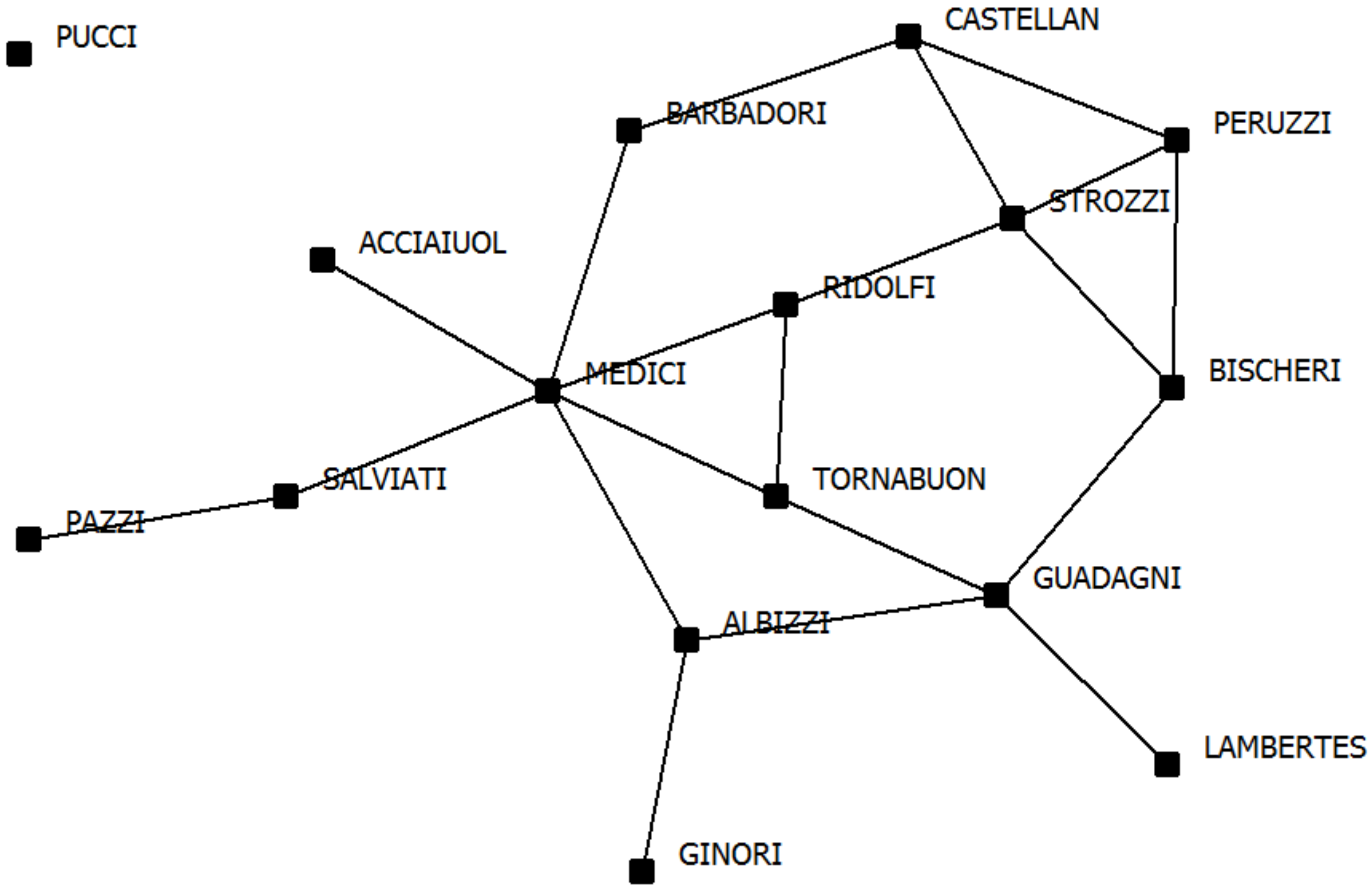
Adjacency matrix

- Can represent a network as a node-by-node matrix
  - Typically 1s and 0s, could be strengths of tie



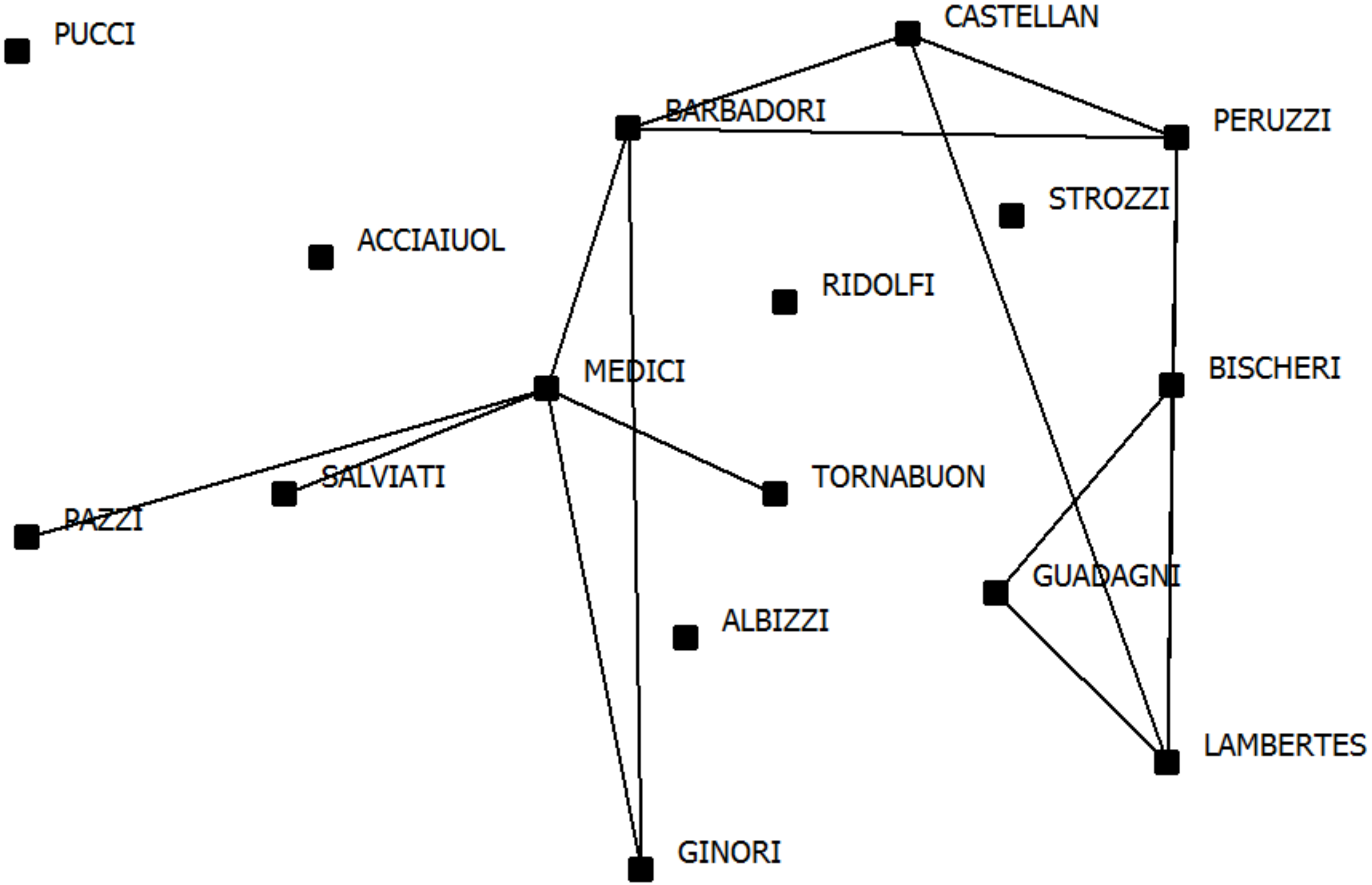
	a	b	c	d	e	f
a		1	0	0	0	0
b	1		1	1	0	0
c	0	1		1	0	0
d	0	1	1		1	0
e	0	0	0	1		1
f	0	0	0	0	1	

Marriage ties between families



Padgett & Ansell (1991). Marriage ties among Florentine families during the Renaissance

Business ties between families

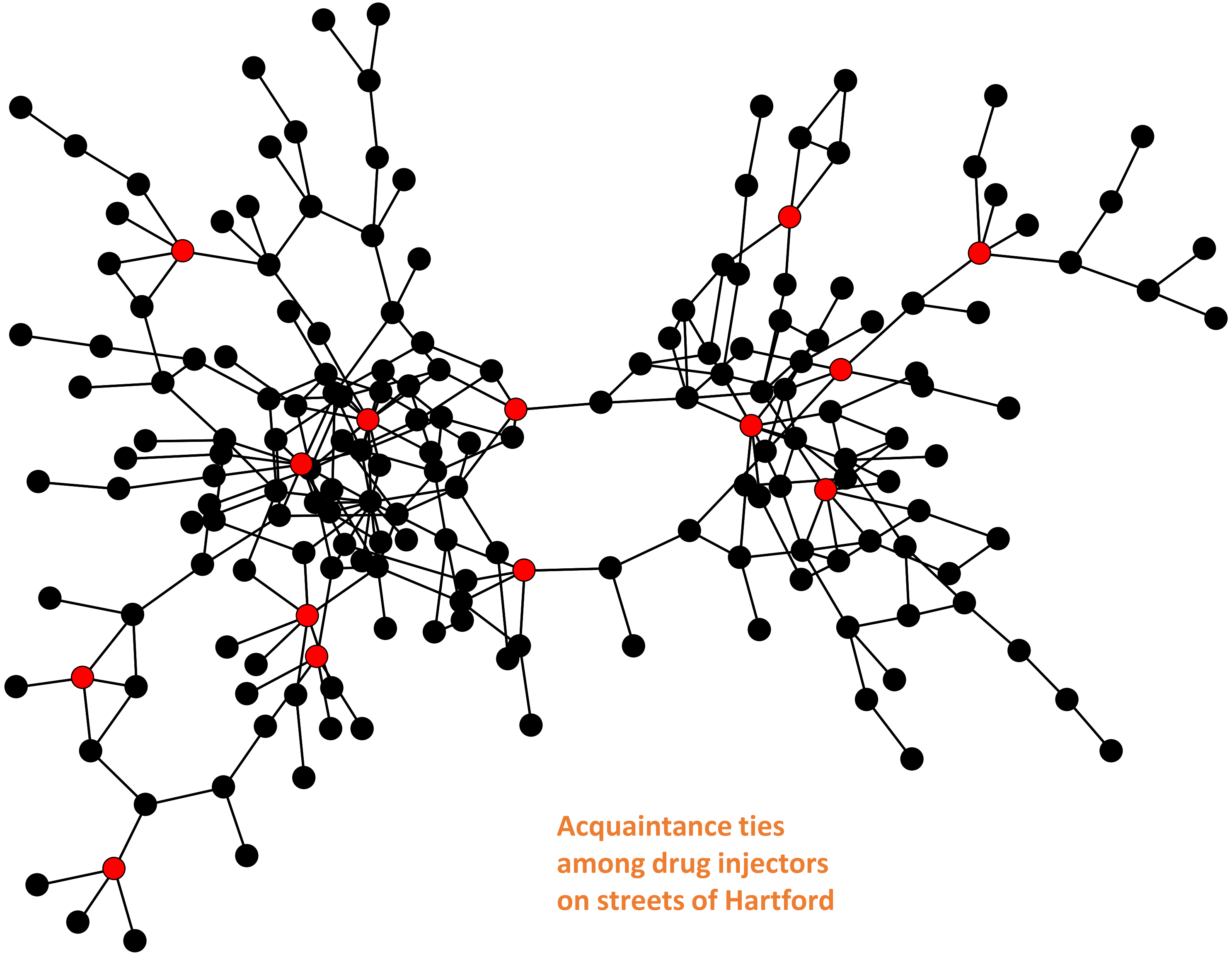


Dyadic variables

- A given type of relation, such as marriage, can be seen as a dyadic variable that describes the relationship between every pair of nodes
- A dyadic variable assigns a value to each pair of nodes

Dyad	Married	Business
ACCIAIUOLI-GUADAGNI	0	0
GUADAGNI-STROZZI	0	0
PUCCI-STROZZI	0	0
BISCHERI-SALVIATI	0	0
ACCIAIUOLI-GINORI	0	0
GUADAGNI-RIDOLFI	0	0
MEDICI-TORNABUONI	1	1
CASTELLANI-SALVIATI	0	0
BARBADORI-GUADAGNI	0	0
CASTELLANI-LAMBERTESCHI	0	1
ACCIAIUOLI-ALBIZZI	0	0
GUADAGNI-PUCCI	0	0
LAMBERTESCHI-STROZZI	0	0
MEDICI-PUCCI	0	0

Acquaintance network



Acquaintance ties
among drug injectors
on streets of Hartford

Is this a network?

Book Co-purchasing

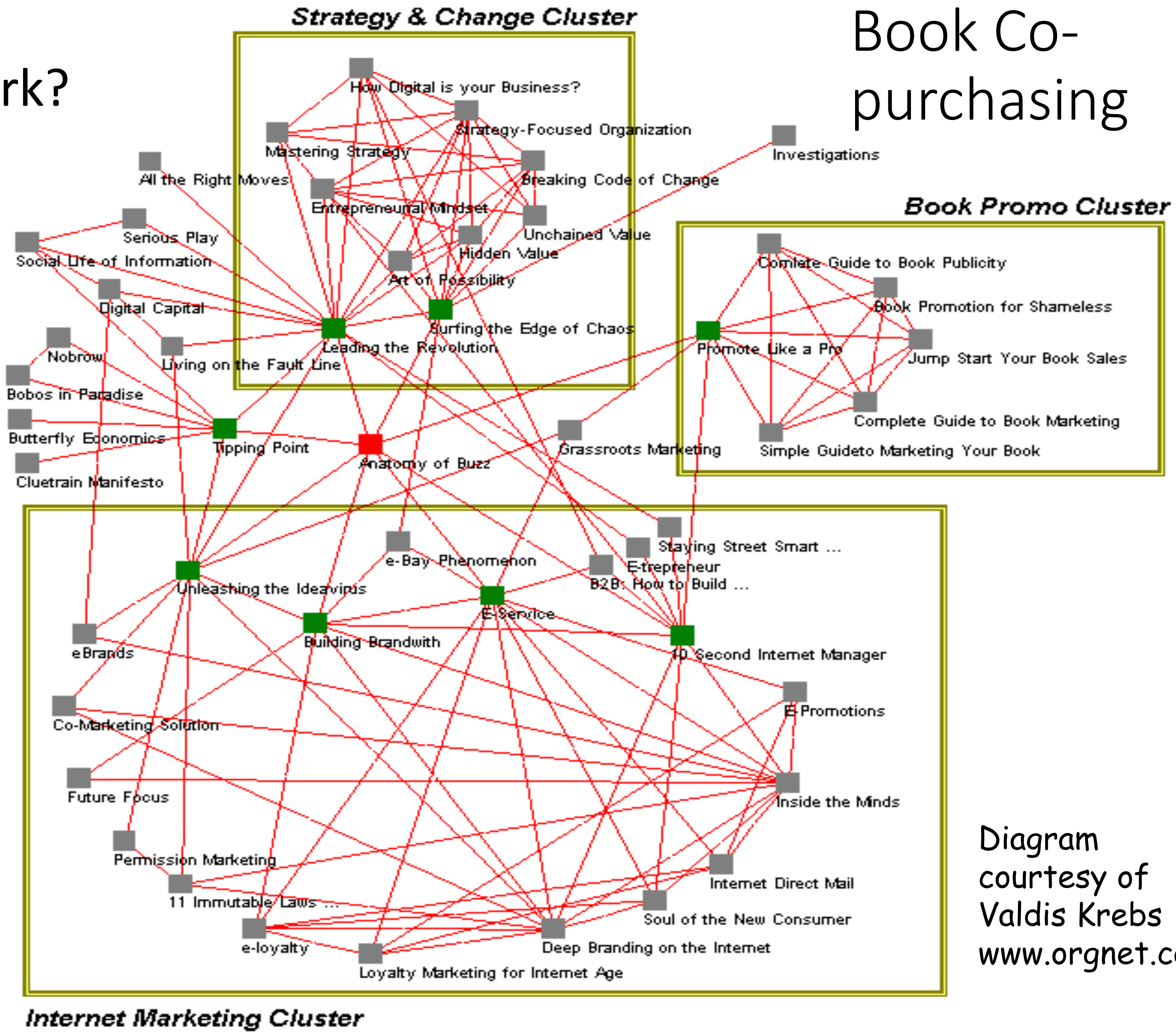
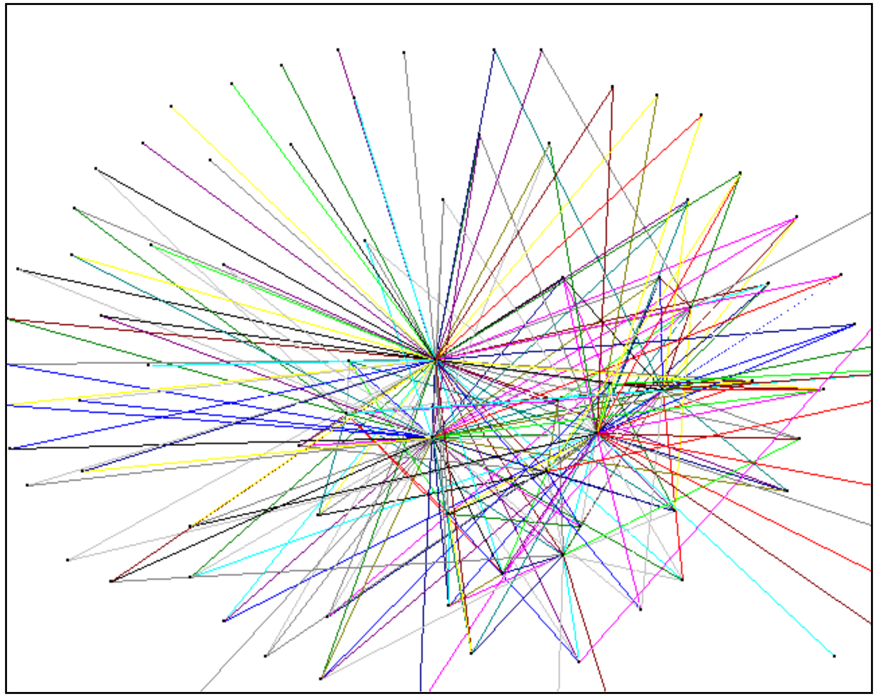


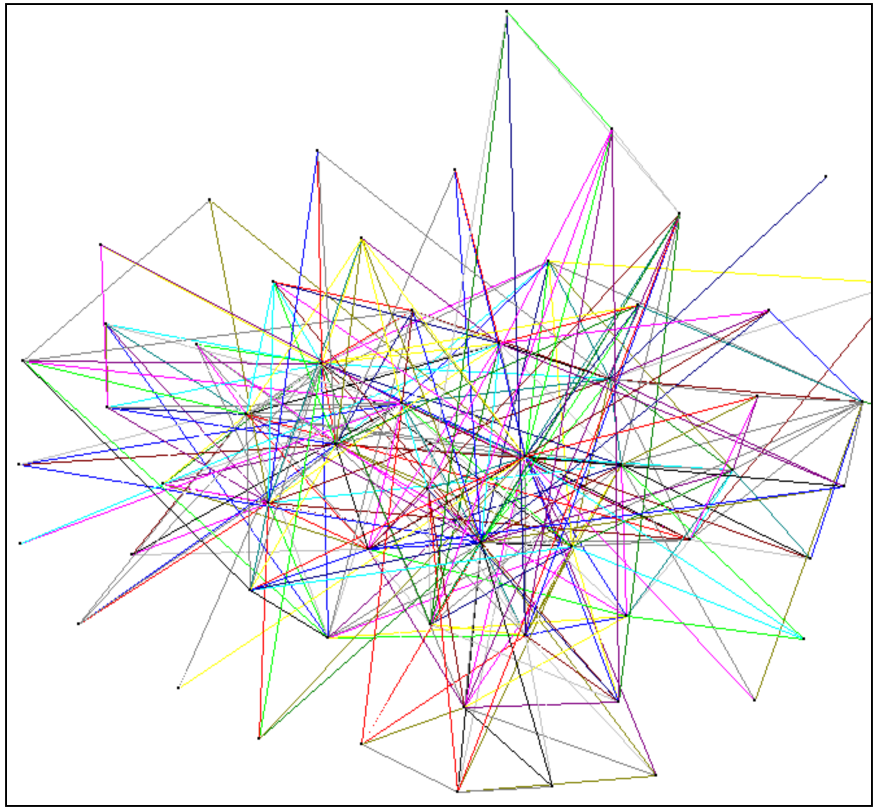
Diagram courtesy of Valdis Krebs www.orgnet.com

Comparing airlines' route structures

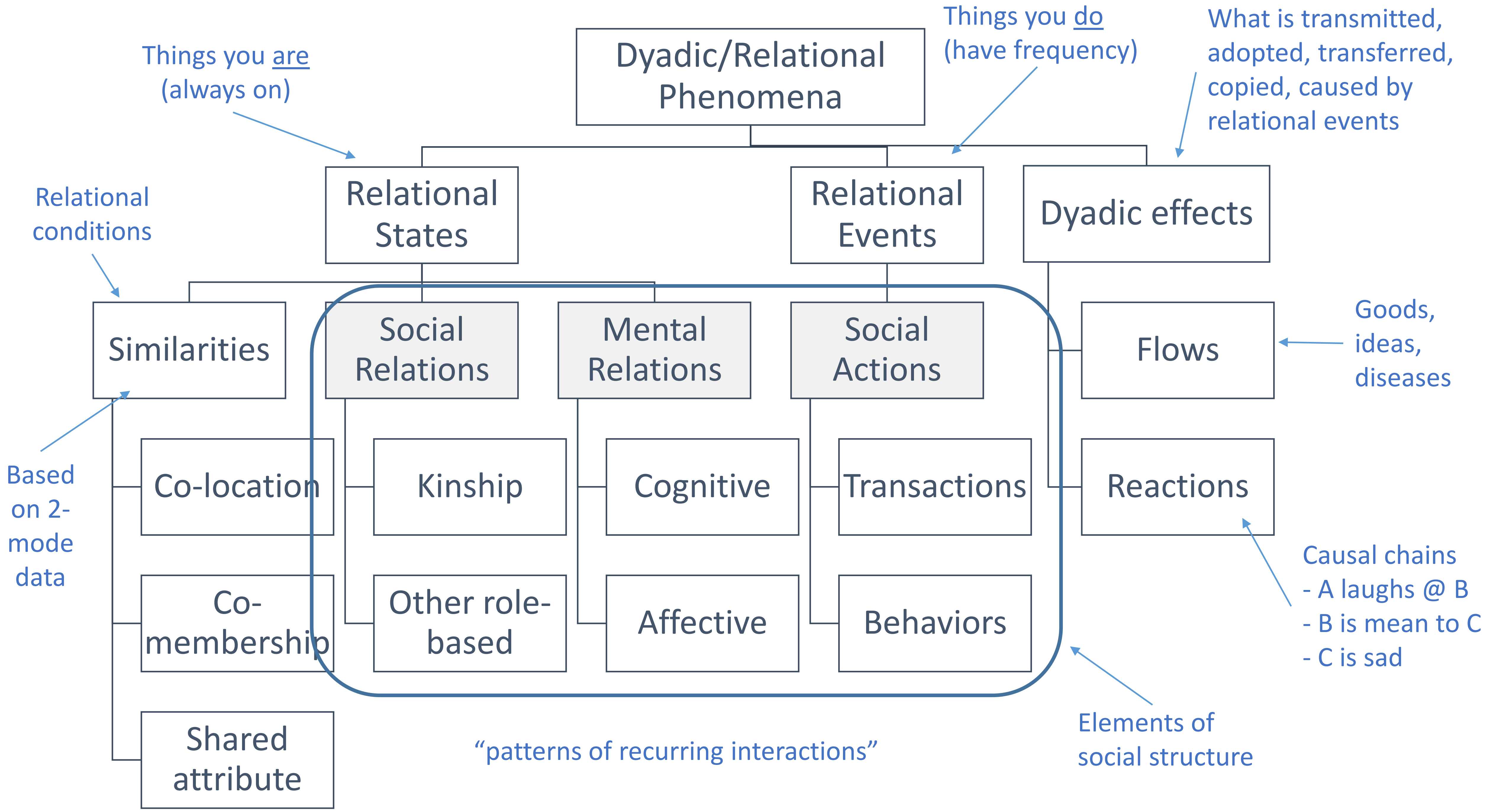
Major Carrier



"Discount" Airline



Note: Route maps defined around one specific hub only
Source: Industry data, BCG analysis


Dyadic/Relational Phenomena
Things you are (always on)
Things you do (have frequency)
What is transmitted, adopted, transferred, copied, caused by relational events
Relational States
Relational Events
Dyadic effects
Relational conditions
Similarities
Social Relations
Mental Relations
Social Actions
Flows
Goods, ideas, diseases
Based on 2-mode data
Co-location
Kinship
Cognitive
Transactions
Reactions
Causal chains
- A laughs @ B
- B is mean to C
- C is sad
Co-membership
Other role-based
Affective
Behaviors
Shared attribute
"patterns of recurring interactions"
Elements of social structure

Entailed interactions

- Friendship carries with it certain norms about how the friends will behave toward each other
  - Rights and obligations
  - Expectations
- Kinship ties have these too
- Professor / student
- So this means that a given “base relation” entails a variety of interactions
  - And base relations also have a variety of different functions, e.g., material aid, emotional support, advice, etc.

Multiplexity

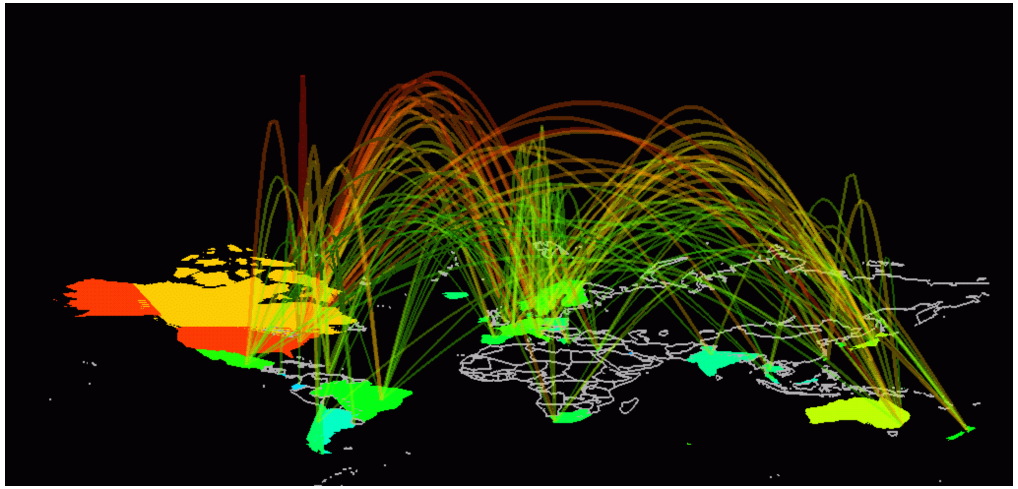
- A given dyad (pair of persons) can be connected by more than one kind of base relation at the same time
  - E.g., both kin and co-worker
- I wouldn't classify being friends and talking often as multiplex
  - Because the base relation entails the talking

Multiplex relationship → MEDICI-TORNABUONI

Dyad	Married	Business
ACCIAIUOLI-GUADAGNI	0	0
GUADAGNI-STROZZI	0	0
PUCCI-STROZZI	0	0
BISCHERI-SALVIATI	0	0
ACCIAIUOLI-GINORI	0	0
GUADAGNI-RIDOLFI	0	0
MEDICI-TORNABUONI	1	1
CASTELLANI-SALVIATI	0	0
BARBADORI-GUADAGNI	0	0
CASTELLANI-LAMBERTESCHI	0	1
ACCIAIUOLI-ALBIZZI	0	0
GUADAGNI-PUCCI	0	0
LAMBERTESCHI-STROZZI	0	0
MEDICI-PUCCI	0	0

Networks: why do we care?

Networks are everywhere



So maybe we should try to understand them?

- A molecule is a network of atoms
- A brain is a network of neurons
- A body contains many networks, including the circulatory system
- Genes form regulatory networks that turn other genes on and off
- Firms are networks of individuals, passing along information, orders and coordinating efforts
- Buildings contain many networks, including heating/cooling, plumbing, electrical
- Economies are networks of firms and other agents buying and selling
- Societies are networks
- Countries contain many networks, e.g., transportation systems, phone systems
- The internet is a network
- Ecosystems are networks of species eating each other, creating environments for each other, etc.

But …

- Networks are also a lens
- We see networks everywhere because we like to think that way
- A network is created any time a researcher says
  - I’m interested in this set of people,
  - And, I define a tie as …. [having the same color hair] [having met before] [etc]
- Don’t want to over-reify networks
- And yet …

Consider the case of AIDS

- 1981 CDC aware of increasing number of cases of opportunistic illnesses like Kaposi's sarcoma
- Virtually all cases were gay men
  - Syndrome initially named Gay-Related Immune Deficiency (GRID)
- Logistic regression of opportunistic illness on being gay
- Proposed mechanism
  - Stigmatized identity causes stress, leading to weakened immune system

Subject ID	Age	Gay	Rare Cancer
1	33	0	0
2	27	0	0
3	89	1	1
4	34	0	0
5	56	1	0
6	23	0	0
7	54	0	0
8	12	1	1
9	45	0	0
10	67	0	0
11	43	1	1
12	21	1	0

Contagion | diffusion | influence mechanisms

Subject ID	Age	Gay	Rare Cancer
1	33	0	0
2	27	0	0
3	89	0	0
4	34	0	0
5	56	1	0
6	23	0	0
7	54	0	0
8	12	1	1
9	45	0	0
10	67	0	0
11	43	1	1
12	21	1	0

Network structure provides backcloth that enables and constrains flows

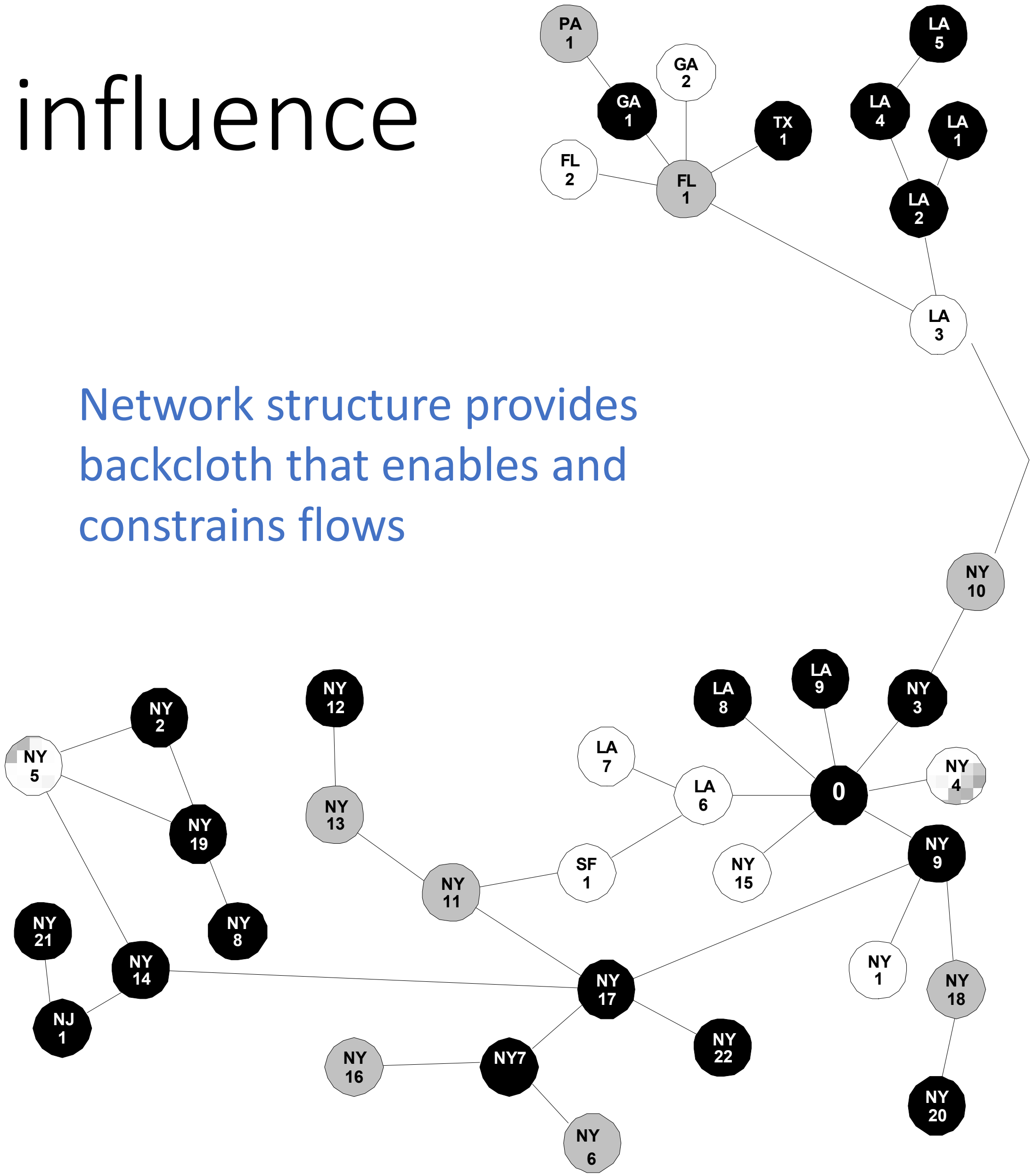
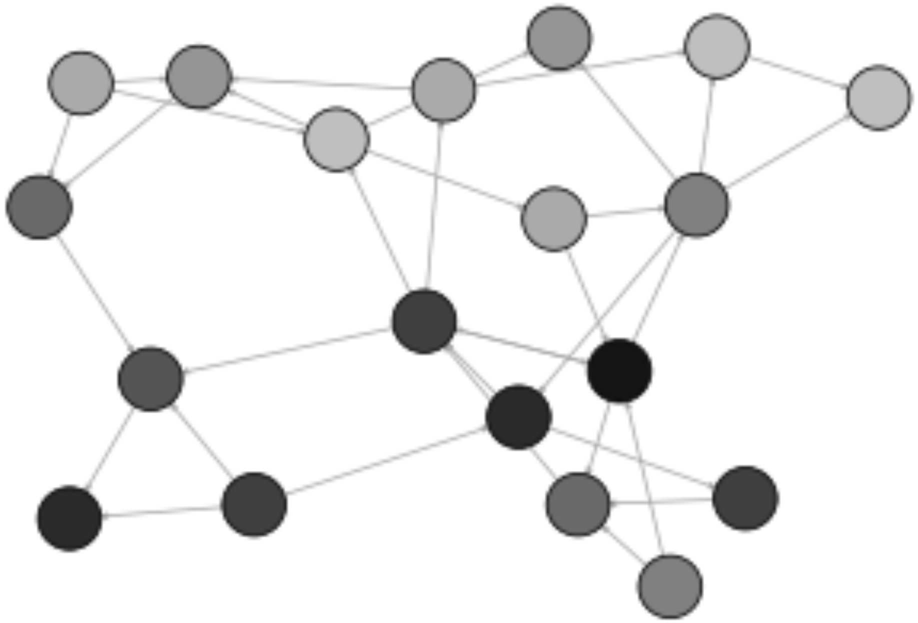
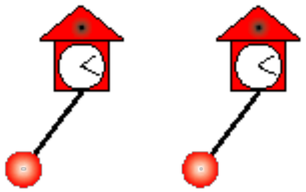


Diagram by Bill Darrow, CDC

Network models of style

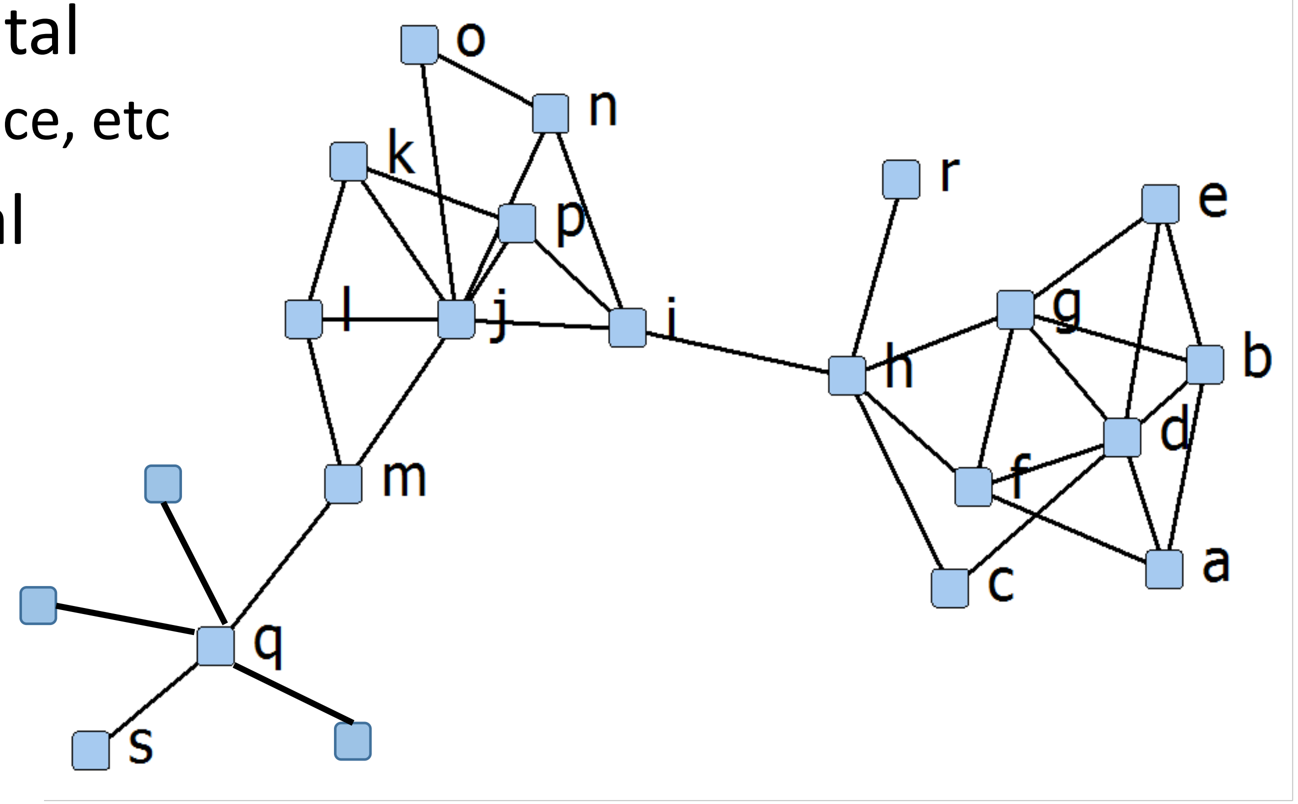


- Why do people …?
  - Wear the clothes they do
  - Speak the way they do
  - Believe the things they do
  - Do things the way they do
  - Etc.
- Partly individual reasons (maximize utility function), but partly contagion/influence from people they know
  - Contagion, diffusion, adoption of innovation, common fate



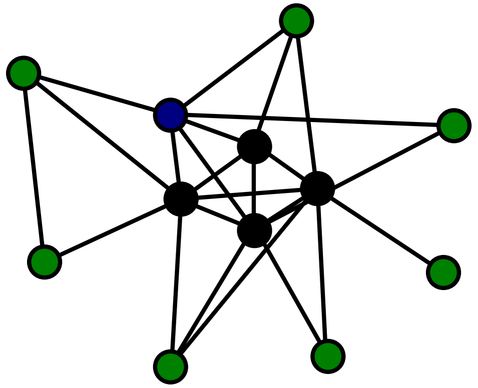
Modeling achievement

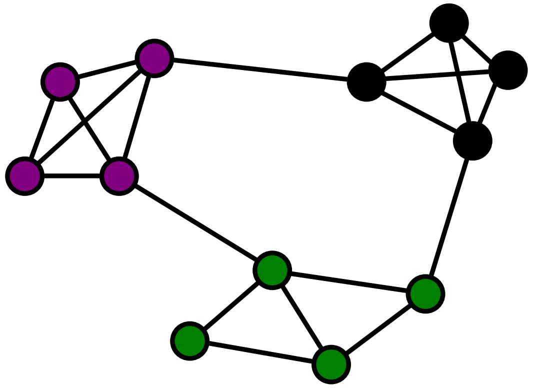
- Why some individuals/organizations are more successful than others
- Standard answer is human capital
  - Motivation, education, intelligence, etc
- Network answer is social capital
  - Position in the network
- Bridging/Brokering positions
  - Access to non-redundant info
  - Freedom of action
  - Combine knowledge from one group to that of another



Levels of analysis -- Organized by most to least number of units

- Dyad level – $O(n^2)$
  - Units are pairs of persons
  - Variables are things like presence of absence of a certain kind of tie between each pair of persons in network
- Node level – O(n)
  - Units are persons
  - Variables are things like the number of friends each person has
- Group/network level – O(1)
  - Units are whole networks (e.g., teams, firms or countries)
  - Variables are things like the density of trust ties, or the average number of degrees of separation between members of the group





Dyad level

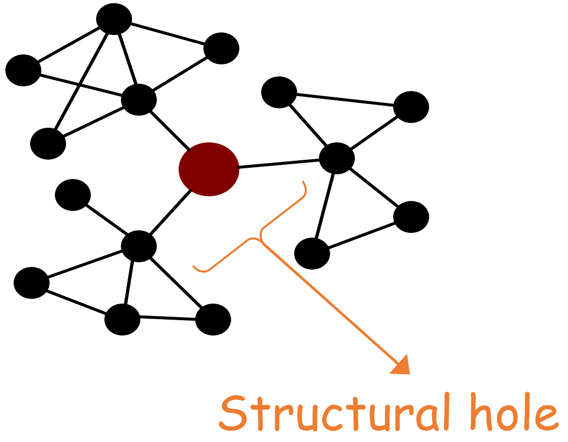
- Raw network data are dyadic
- for each pair of persons we measure
  - whether they have a tie or not (are they friends?)
  - How strong the relationship is (how close are they?)
  - Other aspects of the tie
    - How long have they been friends?
    - How often do they talk?
  - Measurement can undirected or directed
    - Undirected: are they co-workers? If A is coworker of B, then B is coworker of A
    - Directed: advice. Does A give advice to B? If so, maybe B does not give advice to A

Dyad level : antecedents and consequences

- Consequences
  - If A has tie to B, and A knows something, they may tell B, and now both know it
  - So, a consequence of the tie is similarity/homogeneity
    - I have same info as you
    - I adopt same shoes as you
- Antecedents
  - What determines which pair are friends are which are not?
  - Often look to attributes of the individuals
  - So, an antecedent of the tie is similarity

Node level: antecedents and consequents

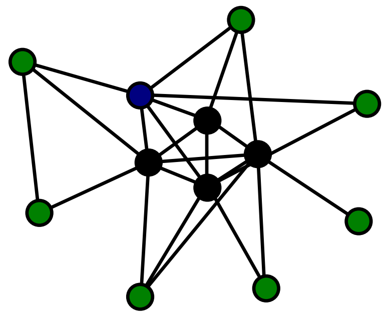
- Consequences
  - Employees with more friends in the higher levels of the organization get promoted earlier and have better raises
  - In management the canonical hypothesis is that managers with more structural holes perform better and get rewarded better

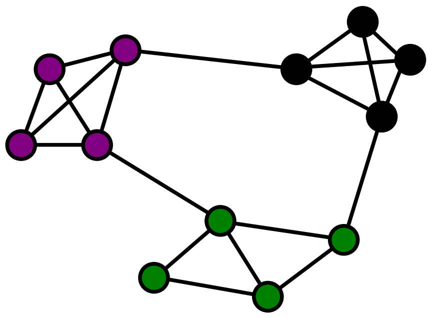


- Antecedents
  - Individuals with more outgoing personalities tend to be more central in the organizational network
  - People with ability to interact productively with diverse kinds of people are more likely to ties to people who are not tied to each other

Group level

- Consequences
  - Teams with more centralized communication networks solve problems more quickly
- Antecedents
  - Teams with greater demographic homogeneity more likely to have core/periphery network structures rather than clumpy structures





Antecedents and consequences

Antecedents

- Socio/cultural/psychological processes that give rise to social ties, interactions, exchanges
  - What determines who is connected to whom?
  - Why do some people have more ties than others?
  - Why does the network have the structure it does?
- **Theory of networks**

Consequences

- Mechanisms that translate ties, positions, structure into outcomes
  - How does the tie between two actors affect what happens between them?
  - How does centrality translate into power?
  - How does network structure determine diffusion speed?
- **Network theory**

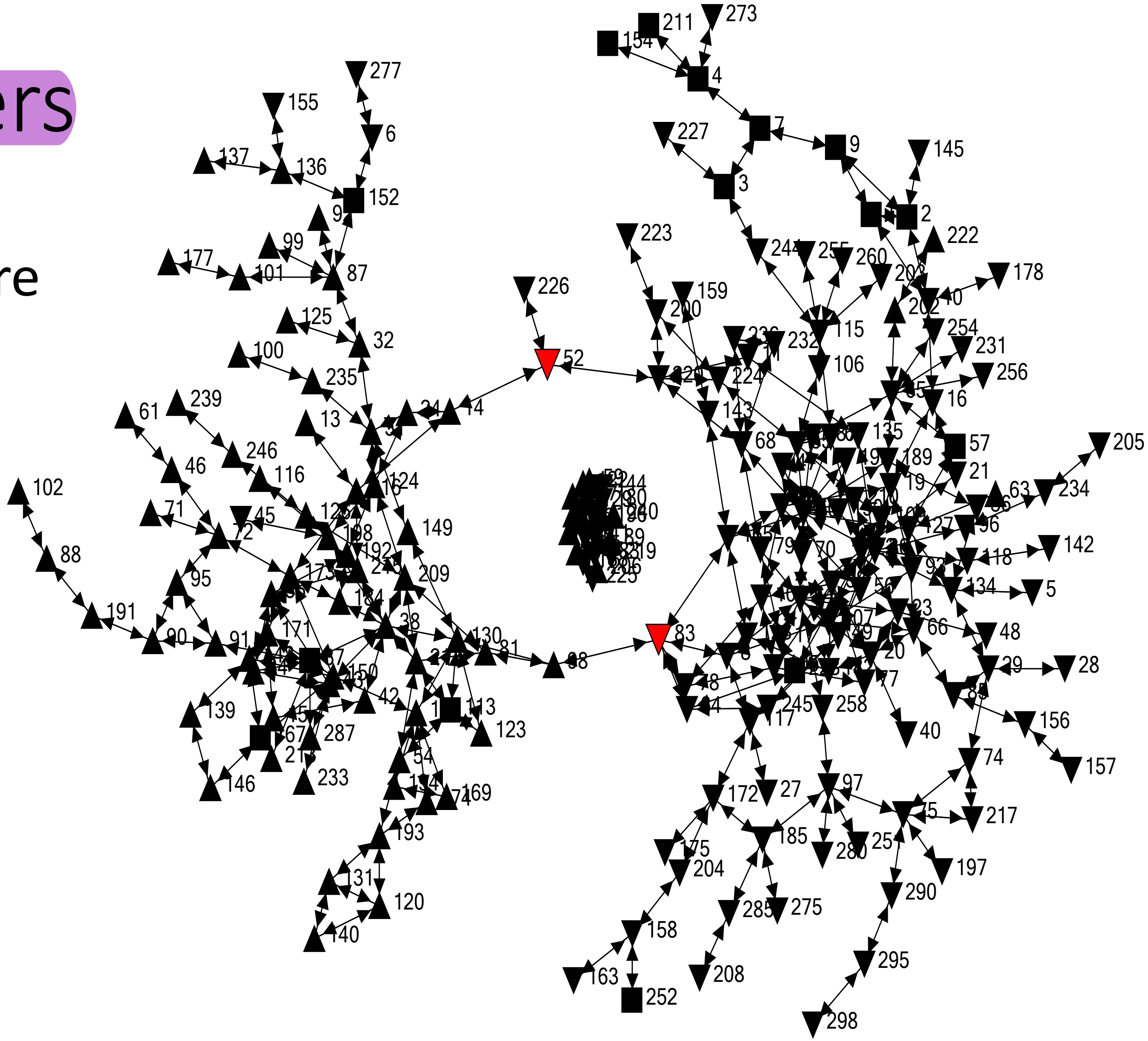
Types of studies

	Dyad Level	Node Level	Group Level
Theory of Networks (Antecedents)	Understanding who becomes friends with whom	Explaining why some people are more liked than others	Explaining why some groups have more centralized network structures
Network Theory (Consequences)	Predicting similarity of opinion as a function of friendship	Explaining why some employees rise through the ranks faster than others as a function of social ties	Predicting team performance as a function of structure of trust network within team

Characteristics of network thinking

Structure matters

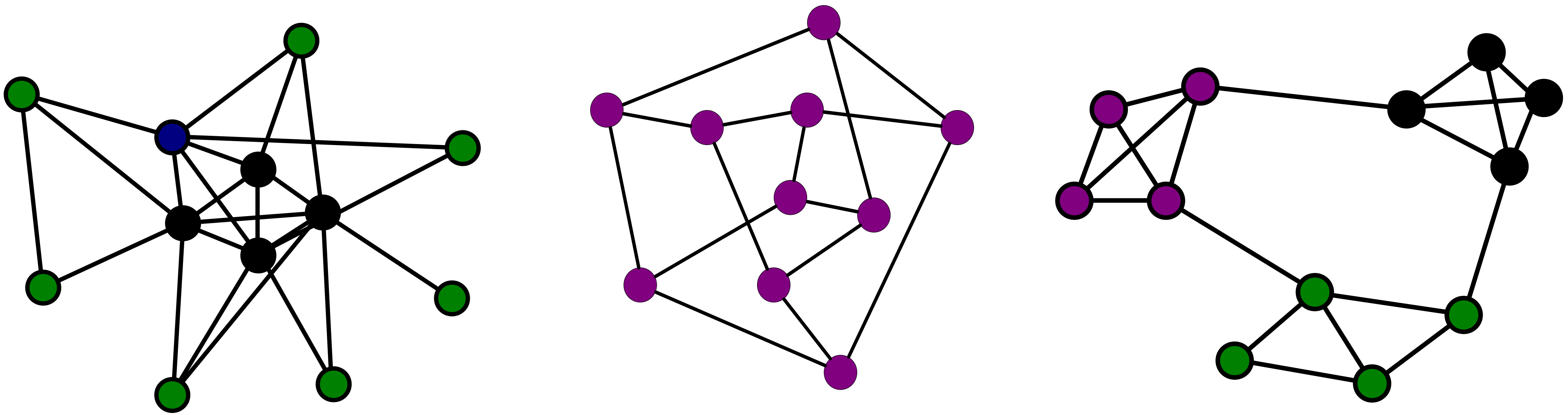
- This is a fragile structure easily broken up



■ whites
▲ african-american
▼ puerto-rican

Which networks are good for what?

- Consequences of these structures for the organization and for nodes

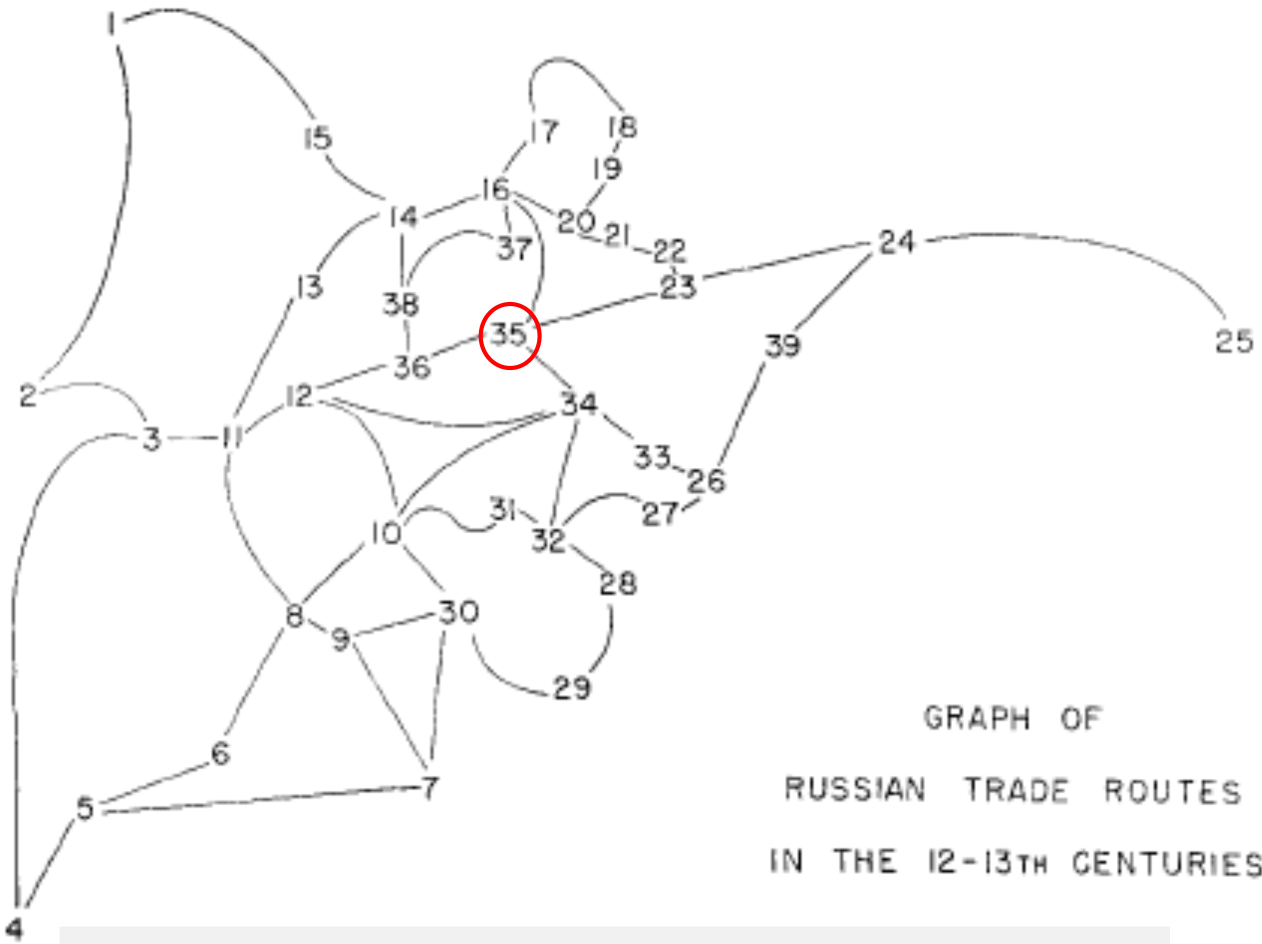


Position matters: the emergence of Moscow

- Pitts (1979) study of 12th century Russia and the later emergence of Moscow
- Why did Moscow come to dominate?
    - Great man theory
    - Resource richness

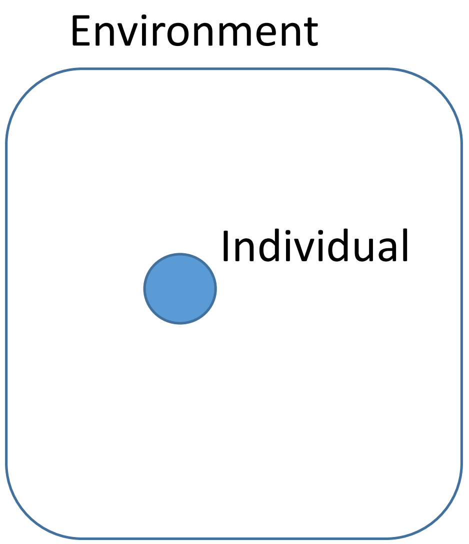
Position matters

- Rivers enable trade between city-states
  - System of rivers creates network of who can trade directly and indirectly with whom
  - What happens in the network is a function of global paths and position
  - Moscow very high in betweenness centrality

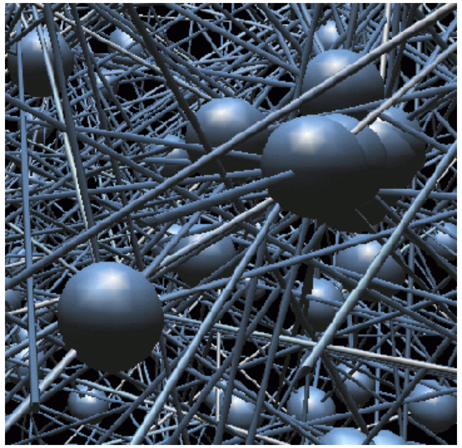


Nodes have high betweenness to the extent they are along the shortest paths between pairs of nodes

SNA as open systems perspective



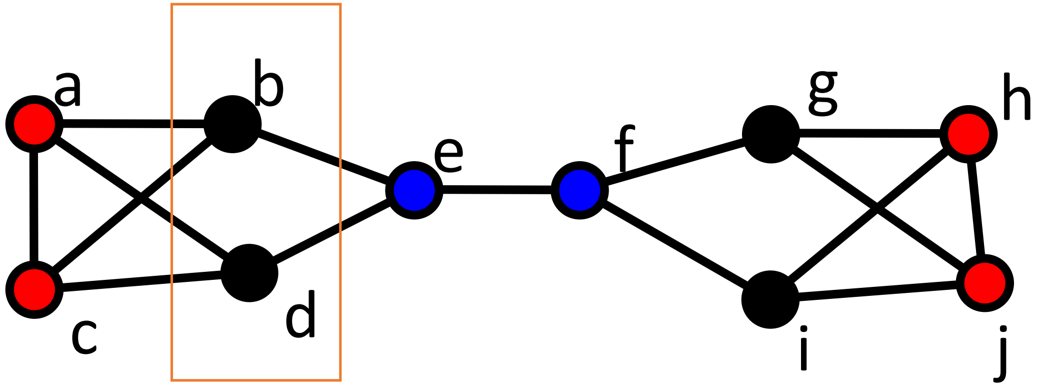
- Importance of an individual's environment
  - To explain individual outcomes, must take into account the node's social environment in addition to internal characteristics
  - In SNA, the environment is conceptualized as network
  - An emphasis on structure relative to agency
  - Consistent with an open systems perspective
- The contrast is with an essentialist/dispositional perspective
  - Predict individual's outcomes using other characteristics of the individual
  - Employee's success a function of ability and motivation



We are all embedded in a thick web of relations

Environment as location in network

- Many fields have concept of environment affecting the individual
  - Turbulent/differentiated environments in organizational theory
- In networks, the environment is conceptualized as other agents
- And these agents are connected to each other and to ego in a particular pattern/structure



Traits versus environment

- Traditionally, social science has focus on attributes of individuals to predict individual outcomes
  - Income as a function of education
  - Essentialist, dispositional, closed system perspective
- SNA looks not only at your own attributes, but also the attribs of the people in your life

Variables (attributes)

Cases (entities)

	Age	Sex	Education	Income
1001				
1002				
1003				
1004				
1005				
…				

Fundamentals of Network Analysis

- **Data** structure
- **Matrix** Algebra
- Set and **graph** theory

Defining & Describing a network

- In social network analysis, we draw on two major areas of mathematics regularly:
    - **Matrix Algebra**
        - Tables of numbers
        - Operations on matrices enable us to draw conclusions we couldn't just intuit
    - **Graph Theory**
        - Branch of discrete math that deals with collections of ties among nodes and gives us concepts like paths

Network vs. Case Perspective

- One of the biggest differences between the SNA perspective and more traditional social science perspectives is the nature of the data
  - Instead of individual cases, where we collect the same information for a bunch of people
  - Here, we collect information about the interaction of pairs of people

Mainstream Logical Data Structure

- 2-mode rectangular matrix in which rows (cases) are entities or objects and columns (variables) are attributes of the cases
- Analysis consists of correlating columns
  - Emphasis on explaining one variable

ID	Age	Education	Salary
1			
2			
3			
4			

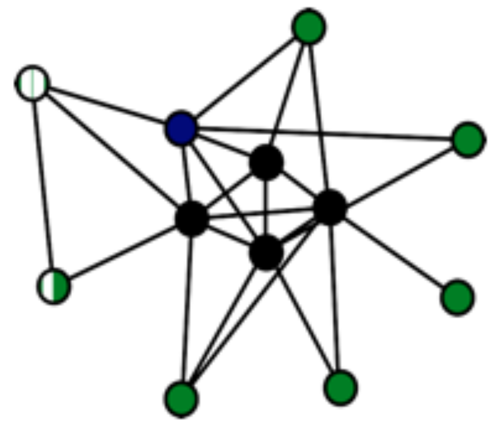
Network Logical Data Structures

Friendship

	Jim	Jill	Jen	Joe
Jim	-	1	0	1
Jill	1	-	1	0
Jen	0	1	-	1
Joe	1	0	1	-

Proximity

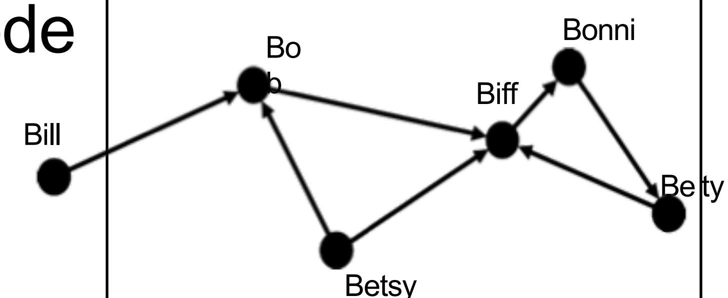
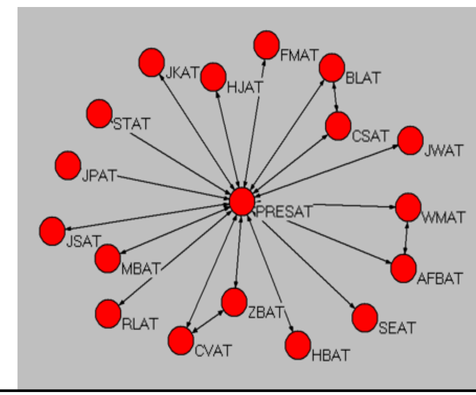
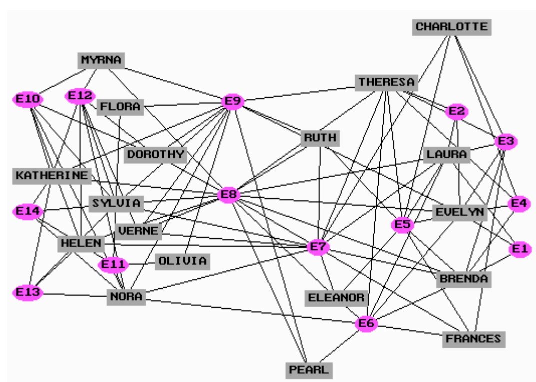
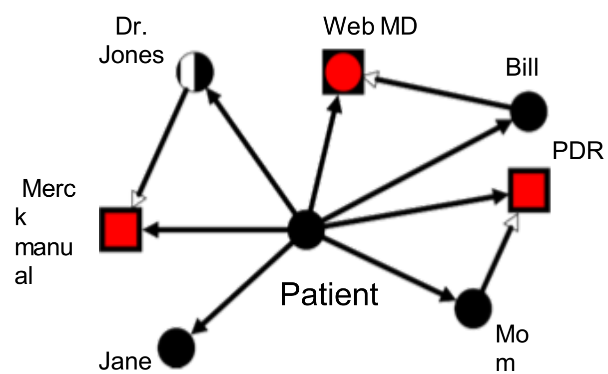
	Jim	Jill	Jen	Joe
Jim	-	3	9	2
Jill	3	-	1	15
Jen	9	1	-	3
Joe	2	15	3	-



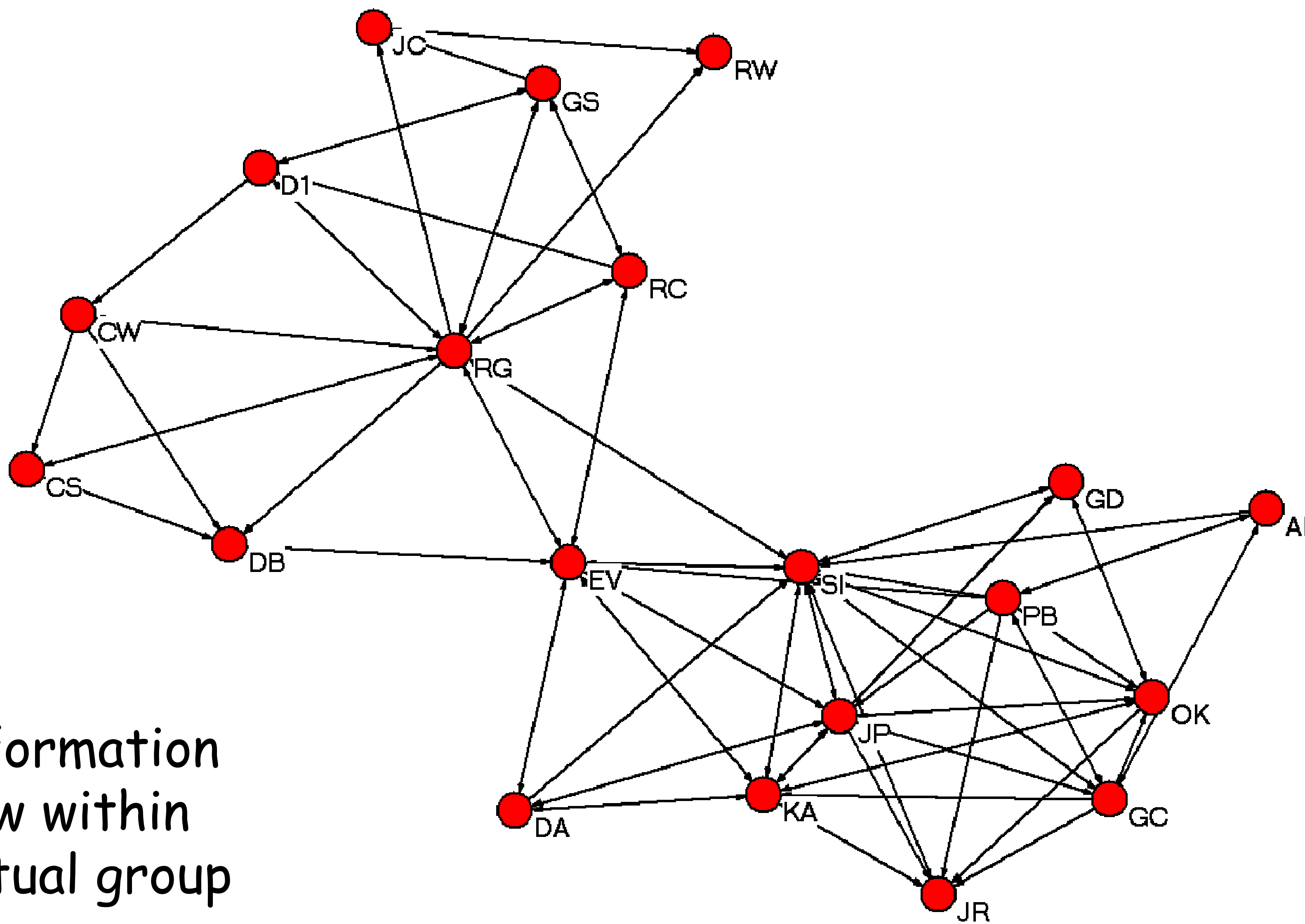
- Multiple relations recorded for the same set of actors
- Each relation is a variable
  - variables can also be defined at more aggregate levels
- Values are assigned to pairs of actors
- Hypotheses can be phrased in terms of correlations between relations
  - Dyadic-level hypotheses

Network Representation

Kinds of Network Data

<table>
<tr><th></th><th>Complete</th><th>Ego</th></tr>
<tr><td>1-mode</td><td>
</td><td>
</td></tr>
<tr><td>2-mode</td><td>
</td><td>
</td></tr>
</table>

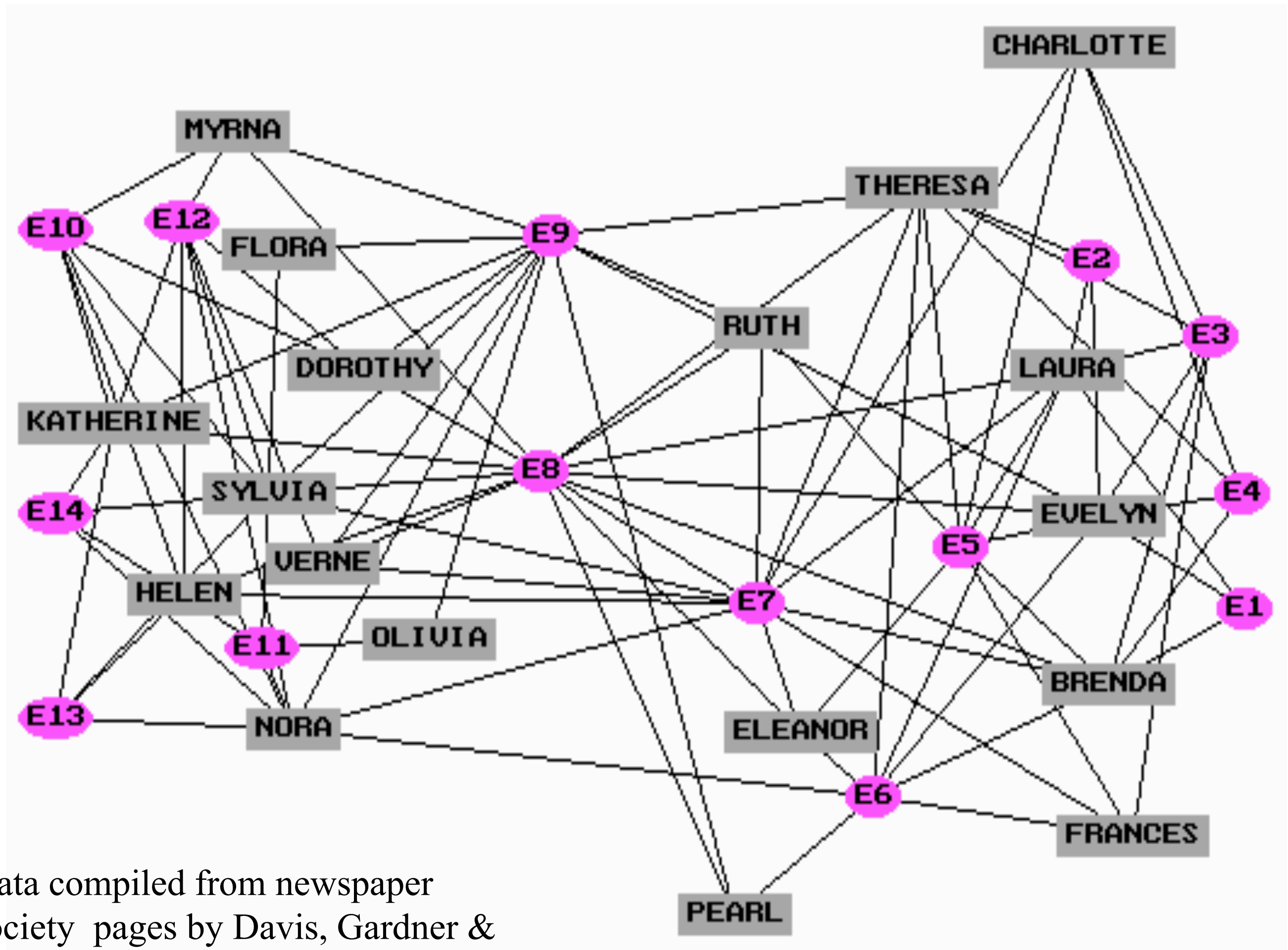
1-mode Complete Network



Information flow within virtual group

Data collected by Cross

2-mode Complete Network

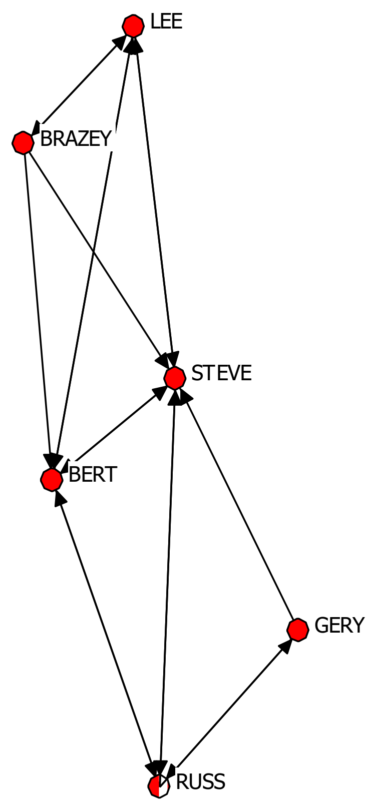


Data compiled from newspaper society pages by Davis, Gardner & Gardner

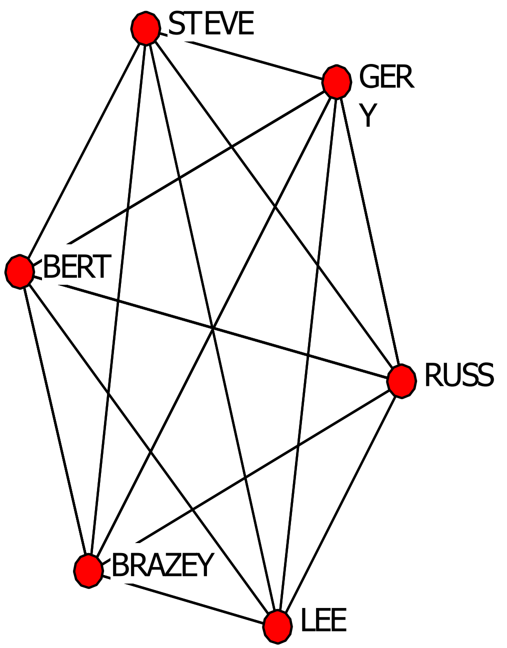
Complete Network Data vs. Complete Graph

- The term “Complete Network Data” refers to collecting data for/from all actors (vertices) on the graph
  - The opposite if Ego-Network or Ego-Centric Network data, in which data is collected only from the perspective an individual (the ego)
- The term “Complete Graph” refers to a graph where every edge that could exist in the graph, does:
  - For all i, j (j>i), v(i,j) = 1

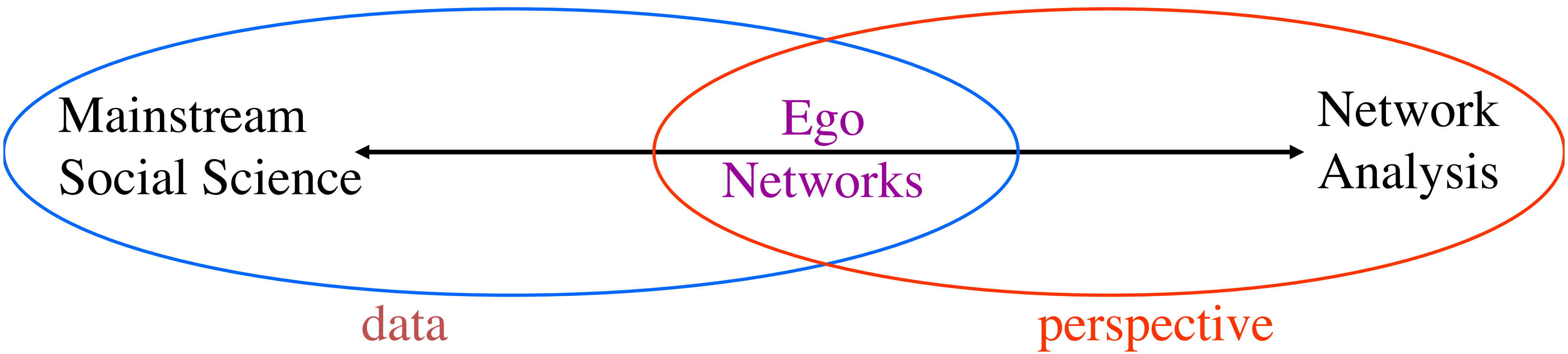
Complete Network Data



Complete Graph



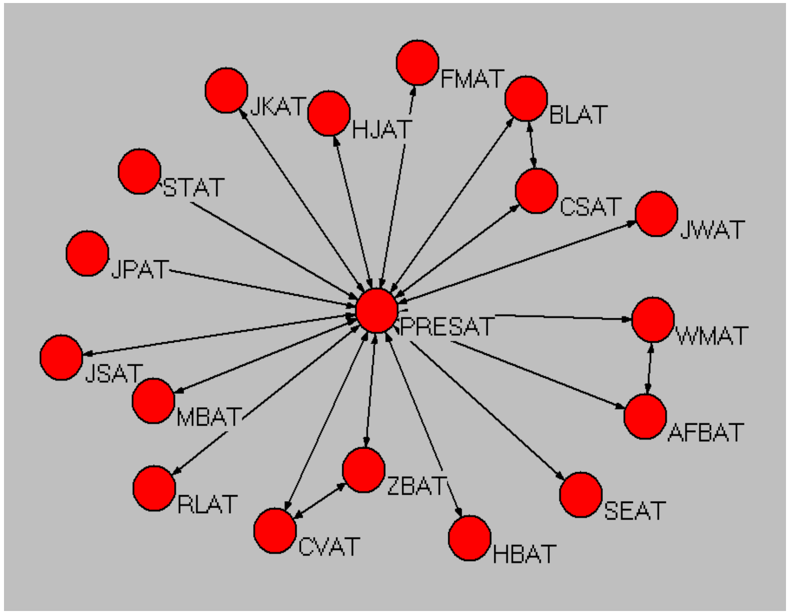
Ego Network Analysis

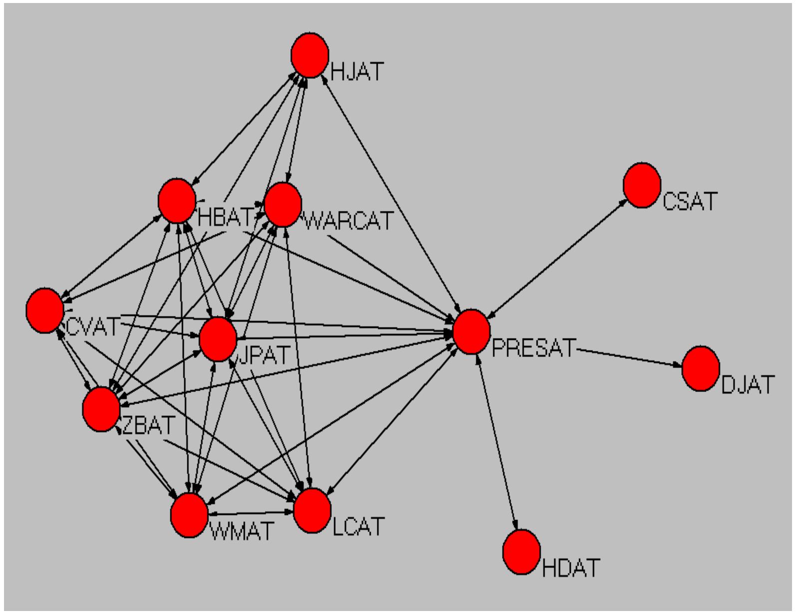


- Combine the perspective of network analysis with the data of mainstream social science

1-mode Ego Network

Carter Administration meetings



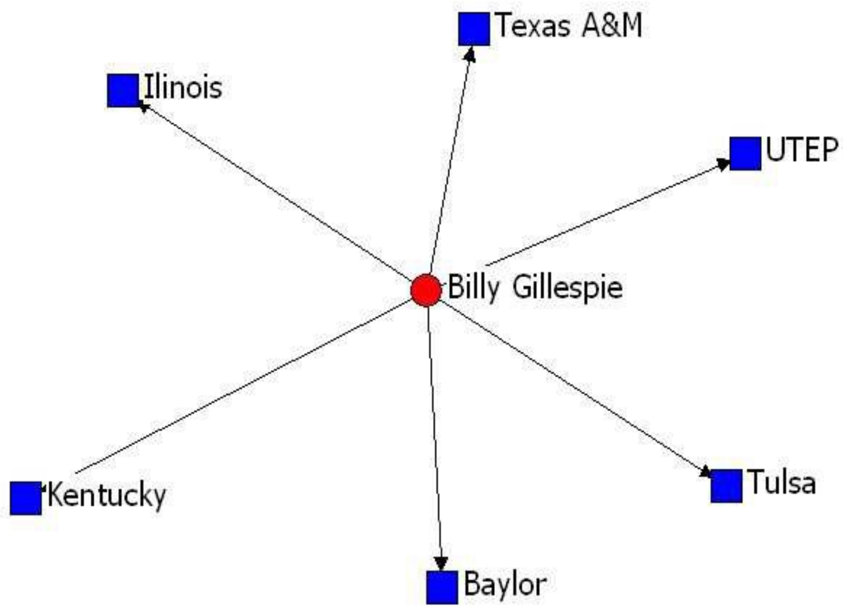


Year 1

Data courtesy of Michael Link

Year 4

2-mode Ego Network



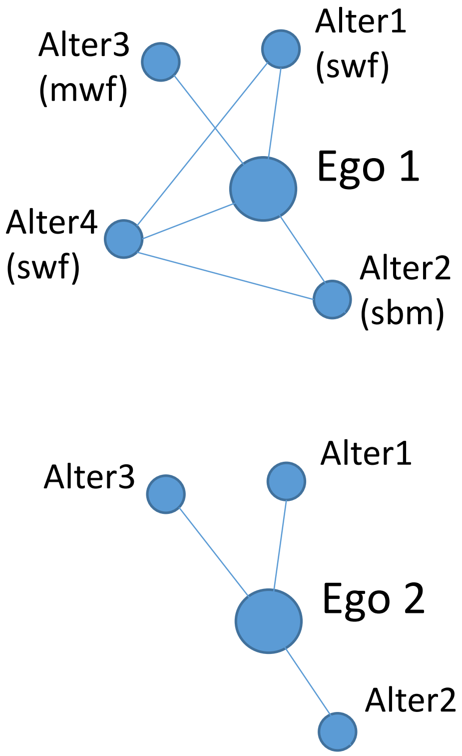
Research designs

Whole network / sociocentric design

- Start with a set of people (typically a “natural” group such as a gang or a department)
- Collect data on the presence/absence (or strength) of ties of various kinds among all pairs of members of the set
  - Who doesn’t like whom; How frequently each pair of persons have a conversation
  - Typically collected via survey: respondent presented with roster of people to select/rate
- Issues
  - The set of persons needs to be some kind of census – can’t randomly pick sample of 100 persons from the population of all Americans
  - The set can’t be too big
  - Problems with inferential validity – how to generalize results?

Personal network / egocentric design

- Select random sample of respondents/subjects
  - Call them egos
- For each subject, identify the set of persons in that subject's life
  - Call them alters
- For each alter, determine their individual characteristics
  - E.g., ask ego how old the alter is, whether they use drugs, etc.
- For each alter, determine the nature of the relationship with ego
  - E.g., ask ego how often they talk to alter, whether alter is a neighbor, etc.
- For pairs alters, determine their relationships to each other
  - E.g., ask ego whether alter 1 is friends with alter 2, etc.



Issues with personal network design

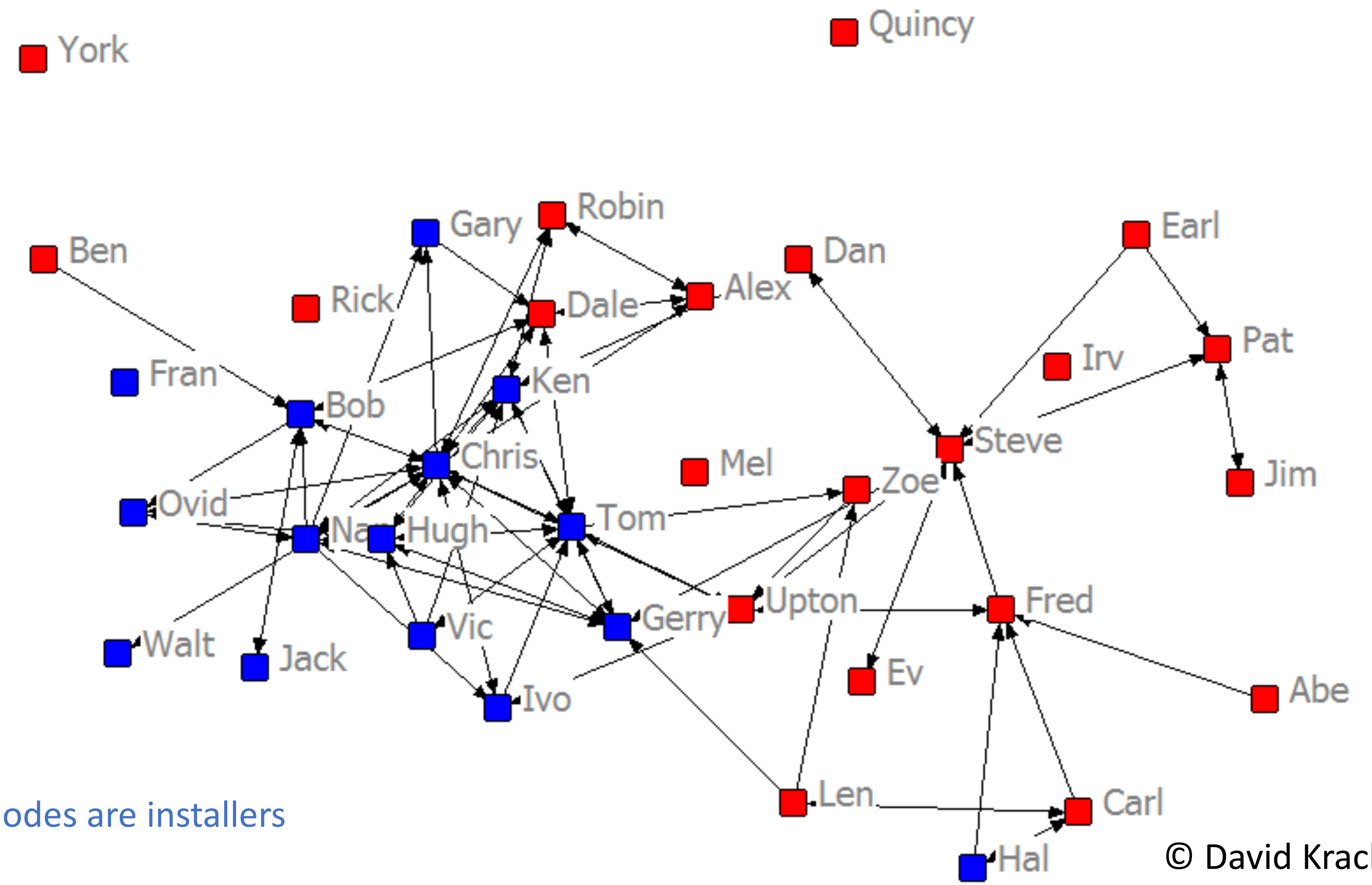
- Can use random samples, enabling generalizability of findings
- Can study very large populations
- Can't say anything about network structure, or position of nodes within the structure
- Typically collected via survey, so all of the information about alters is obtained from ego's perceptions
  - May be inaccurate
  - But maybe it is ego's perception that matters …

Cognitive social structures (CSS) design

- A blend of whole network and personal network designs
- Start with natural group of persons as in whole network design
- Ask each person to indicate not only their own relationship with each other person, but also their perception of the relationships among all pairs of persons
- Result is a perceived network from each member of the network
- Issues
  - Tedious for the respondent – can only be used with small groups
  - Extremely rich data. Can calculate accuracy of each person’s perceptions. Study effects of social perceptions

"Who would this person consider to be a personal friend? Please place a check next to all the names of those people who that person would consider to be a friend of theirs"

Friendship network -- ilas

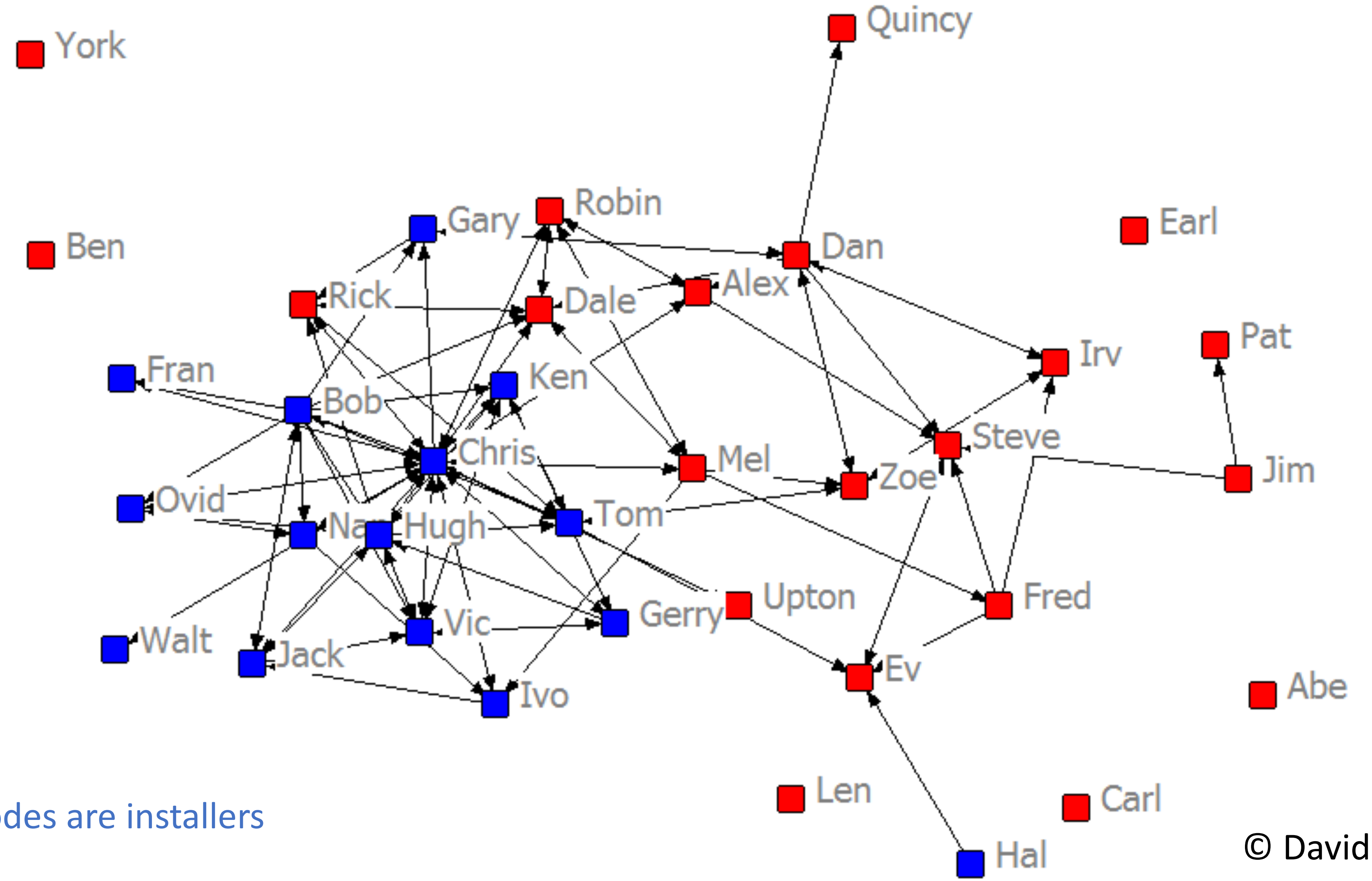


Node	Indeg
Chris	10
Ken	8
Tom	8
Gerry	7
Steve	6
Hugh	6
Bob	5
Dale	5
Nan	5
Fred	4
Ovid	4
Upton	4
Ivo	3
Pat	3
Robin	3
Alex	3
Carl	2
Gary	2
Zoe	2
Dan	2

Blue nodes are installers

"Who would this person consider to be a personal friend? Please place a check next to all the names of those people who that person would consider to be a friend of theirs"

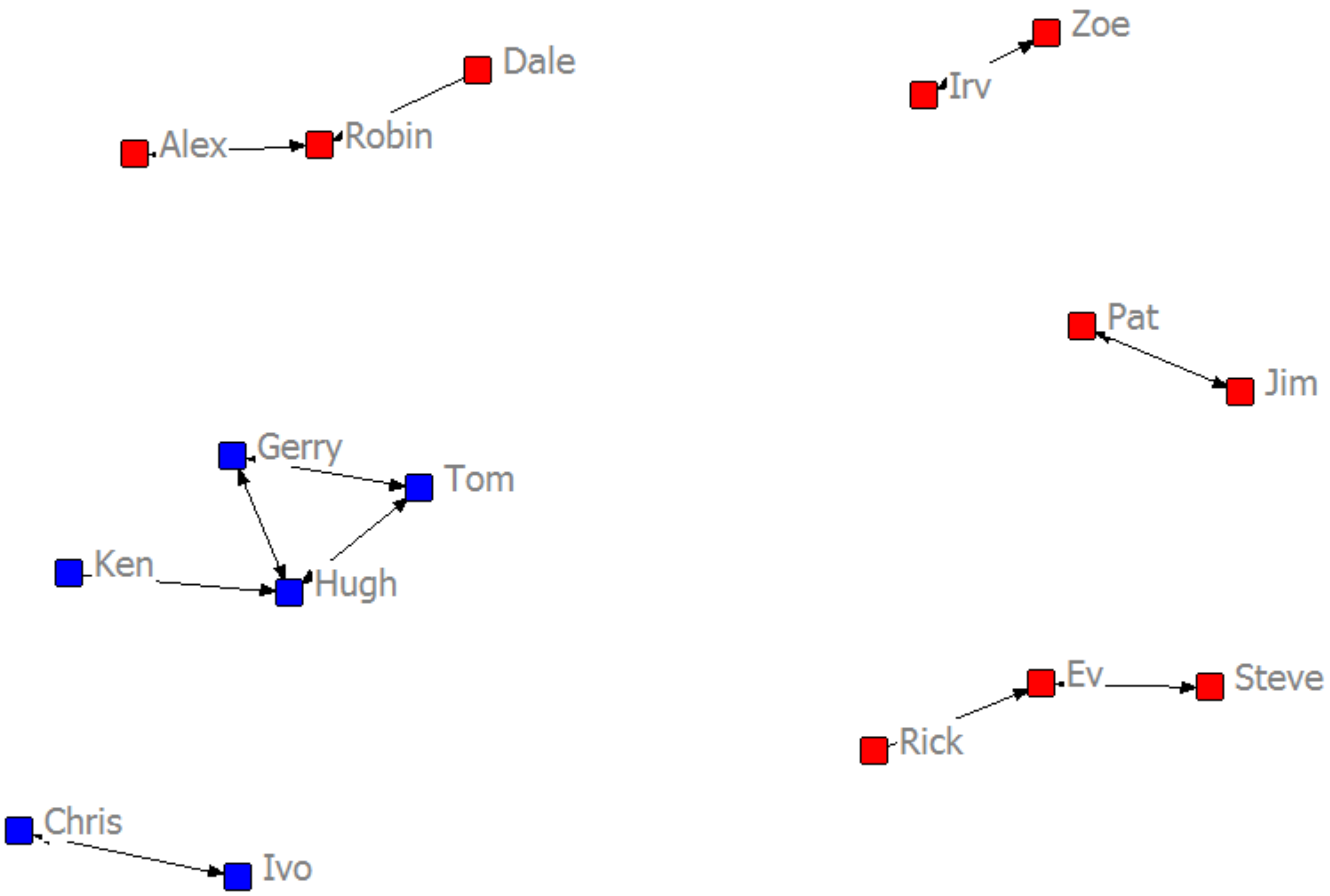
Chris's perception of the friendship network



Blue nodes are installers

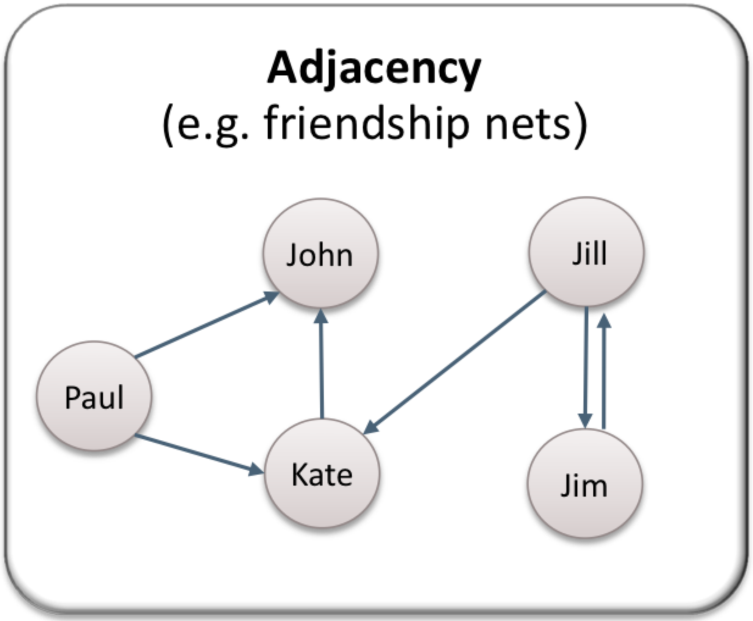
"Who would this person consider to be a personal friend? Please place a check next to all the names of those people who that person would consider to be a friend of theirs"

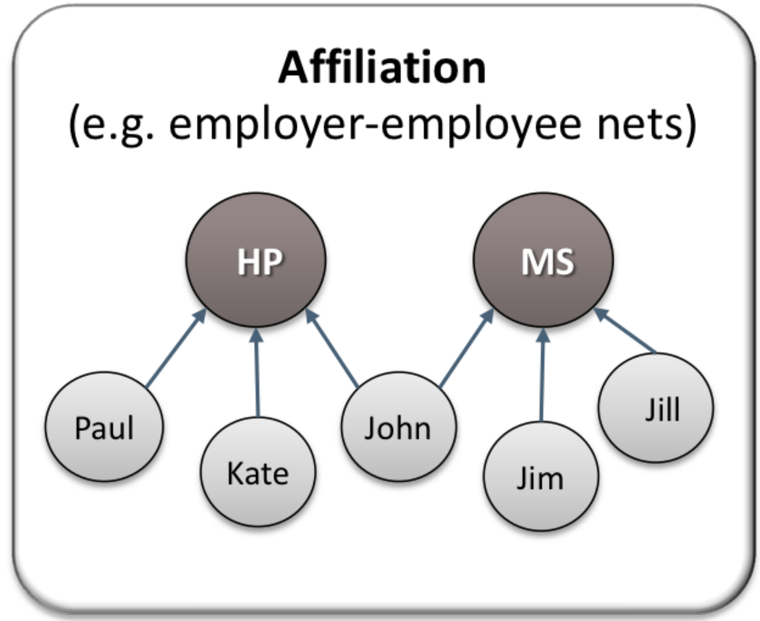
Ev's perception of the friendship network



Representing Networks

representing networks – network modes





representing networks – link types

Binary

- **Link or no link** (1 or 0)

Signed

- **Positive or negative** (+, - or 0)

Valued

- **Weighted links** (each link is assigned a value)

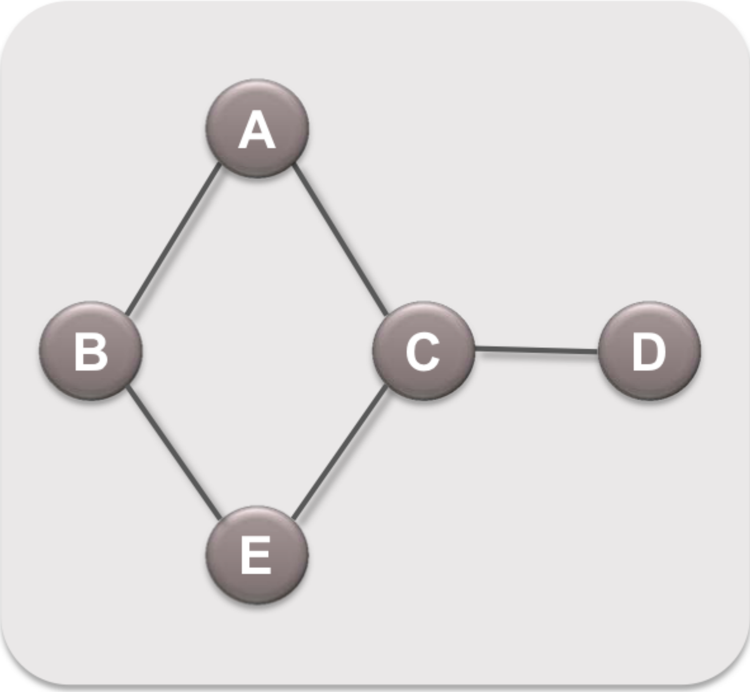
Symmetric

- **Directed vs. undirected** (undirected = symmetric)

Multiplex

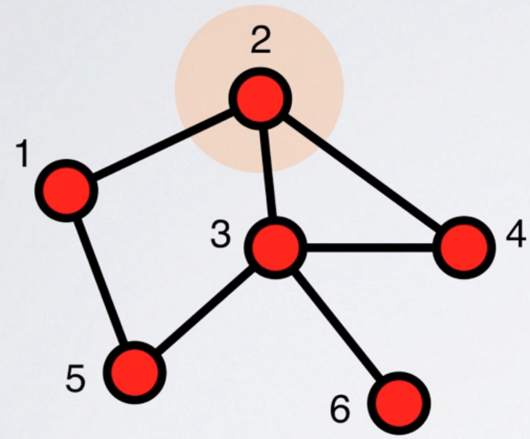
- **Multiplex** (more than one link type)

representing networks – symmetric networks



	A	B	C	D	E
A	0	1	1	0	0
B	1	0	0	0	1
C	1	0	0	1	1
D	0	0	1	0	0
E	0	1	1	0	0

representing networks – simple undirected



undirected

unweighted

no self-loops

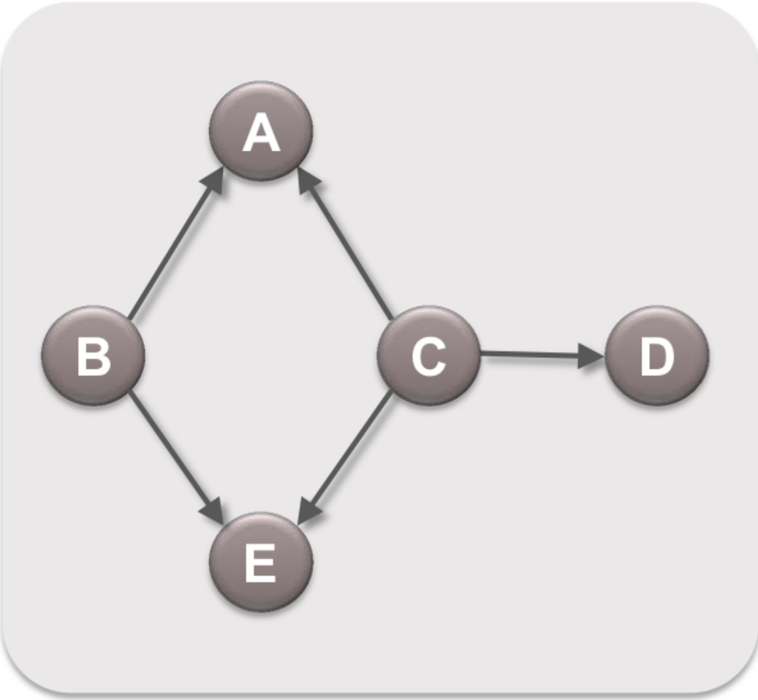
adjacency matrix

A	1	2	3	4	5	6
1	0	1	0	0	1	0
2	1	0	1	1	0	0
3	0	1	0	1	1	1
4	0	1	1	0	0	0
5	1	0	1	0	0	0
6	0	0	1	0	0	0

adjacency list

A
$1 \rightarrow \{2, 5\}$
$2 \rightarrow \{1, 3, 4\}$
$3 \rightarrow \{2, 4, 5, 6\}$
$4 \rightarrow \{2, 3\}$
$5 \rightarrow \{1, 3\}$
$6 \rightarrow \{3\}$

representing networks – directed networks



	A	B	C	D	E
A	0	0	0	0	0
B	1	0	0	0	1
C	1	0	0	1	1
D	0	0	0	0	0
E	0	0	0	0	0

representing networks – directed networks

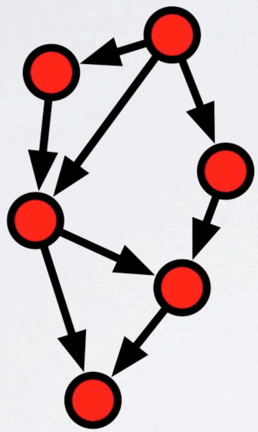
$$A_{ij} \neq A_{ji}$$

citation networks

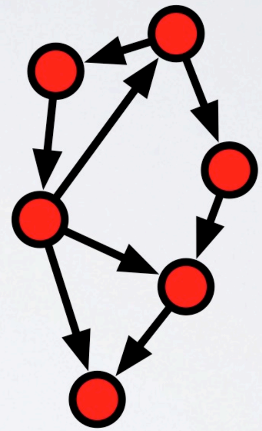
foodwebs*

epidemiological

others?



directed acyclic graph



directed graph

WWW

friendship?

flows of goods, information

economic exchange

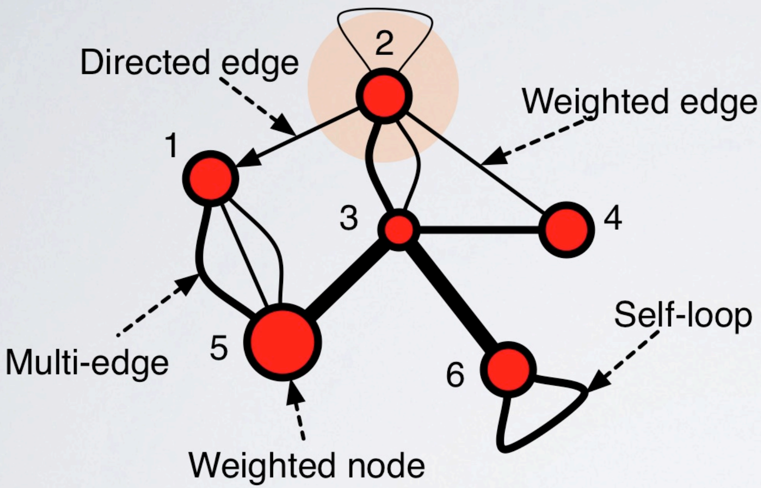
dominance

neuronal

transcription

time travelers

representing networks – complex



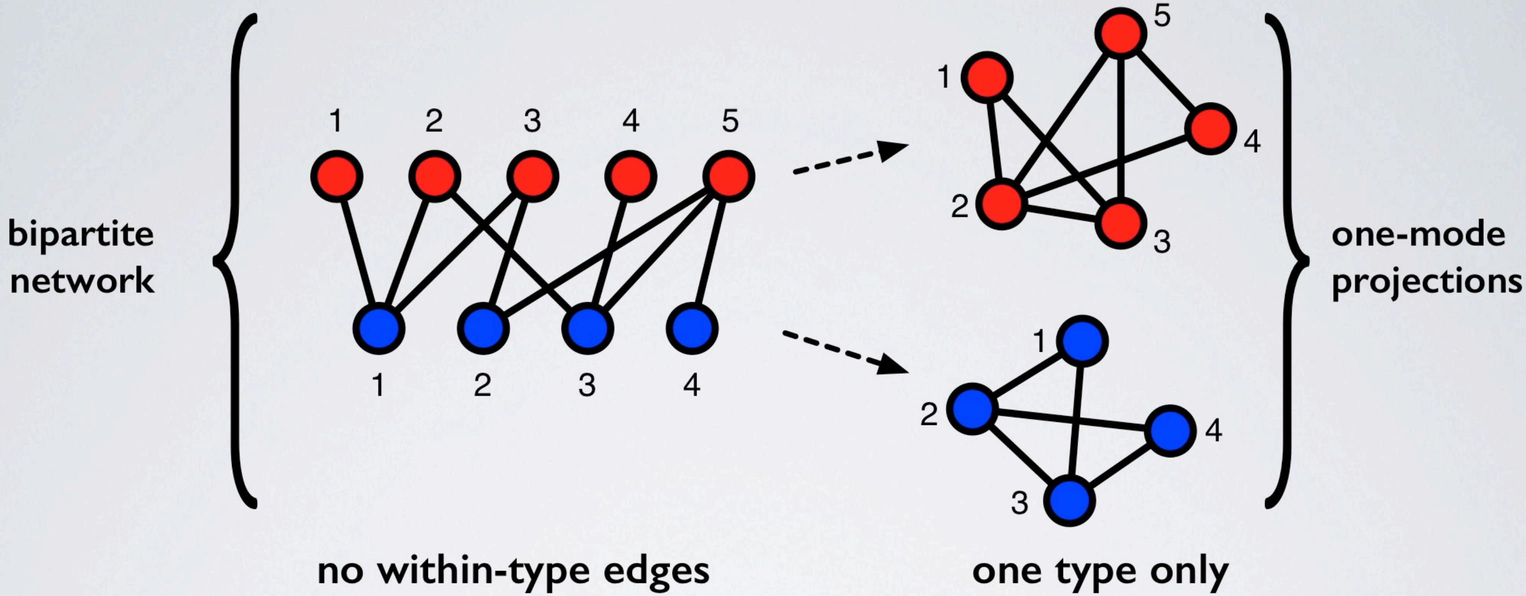
adjacency matrix

A	1	2	3	4	5	6
1	0	0	0	0	$\{1,1,2\}$	0
2	1	$\frac{1}{2}$	$\{2,1\}$	1	0	0
3	0	$\{2,1\}$	0	2	4	4
4	0	1	2	0	0	0
5	$\{1,1,2\}$	0	4	0	0	0
6	0	0	4	0	0	2

adjacency list

A	
1	$\rightarrow \{(5,1),(5,1),(5,2)\}$
2	$\rightarrow \{(1,1),(2,\frac{1}{2}),(3,2),(3,1),(4,1)\}$
3	$\rightarrow \{(2,2),(2,1),(4,2),(5,4),(6,4)\}$
4	$\rightarrow \{(2,1),(3,2)\}$
5	$\rightarrow \{(1,1),(1,1),(1,2),(3,4)\}$
6	$\rightarrow \{(3,4),(6,2)\}$

representing networks – bipartite networks



authors & papers

actors & movies/scenes

musicians & albums

people & online groups

people & corporate boards

people & locations (checkins)

metabolites & reactions

genes & substrings

words & documents

plants & pollinators

representing networks – complex

attributes of

edges	nodes	network
unweighted	metadata / attributes	sparse
weighted	locations / coordinates	dense
signed	state variables	bipartite
undirected		projection
directed		connected
multigraph		disconnected
timestamps		acyclic
		temporal
		multiplex
		hypergraph

Network Data

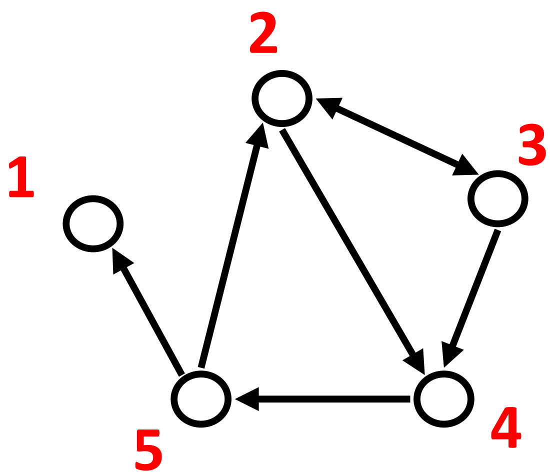
storing network data

1. Adjacency matrix

2. Edgelist

3. Adjacency/node list

1. Adjacency Matrix



$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Issues:

1. Your dataset will likely contain network data in a non-matrix format;
2. Large, sparse networks take way too much space if kept in a matrix format

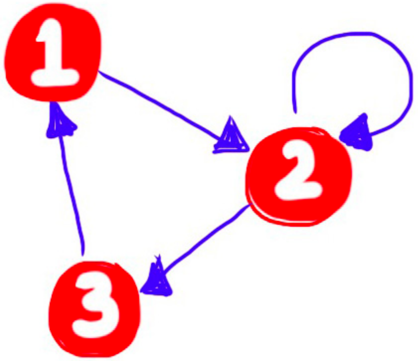
1. Adjacency Matrix

Which adjacency matrix represents this network?

A) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

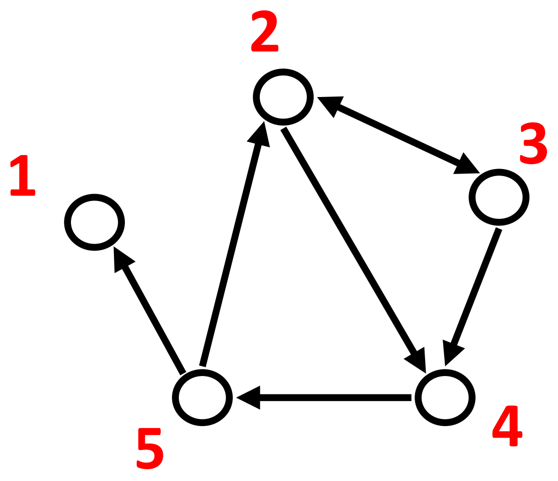
B) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

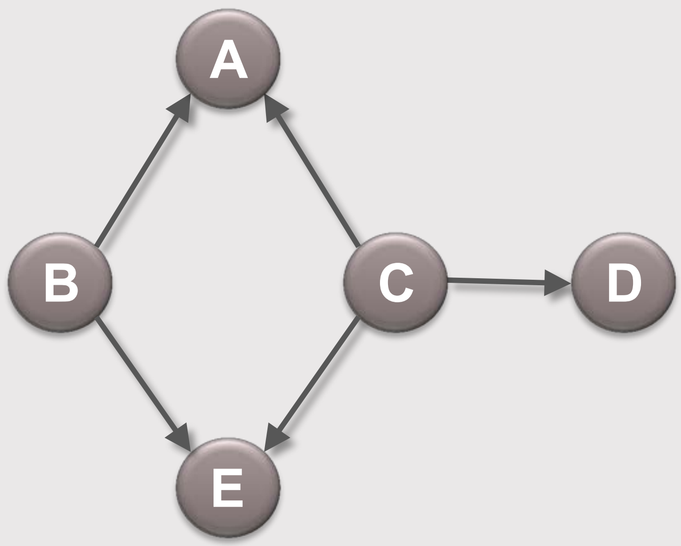


2. <u>Edge list</u>

- Edge list
  - 2, 3
  - 2, 4
  - 3, 2
  - 3, 4
  - 4, 5
  - 5, 2
  - 5, 1



2. Edge List (with weights)

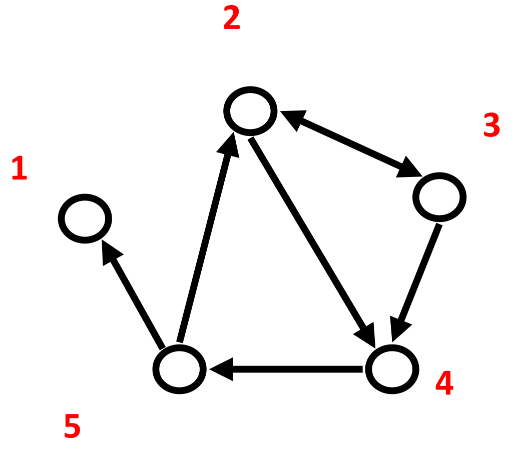


Source	Destination	Weight
B	A	1
B	E	1
C	A	1
C	E	1
C	D	1

Note: Weights are optional.

3. Adjacency list | Node list

- Adjacency list
  - is easier to work with if network is
    - large
    - sparse
  - quickly retrieve all neighbors for a node
    - 1:
    - 2: 3 4
    - 3: 2 4
    - 4: 5
    - 5: 1 2



Matrix Algebra

Matrix Algebra

- Matrix Concepts, Notation & Terminologies
- Adjacency Matrices
- Transposes
- Matrix Operations

Matrices

- Symbolized by a capital letter, like A
- Each cell in the matrix identified by row and column subscripts: a ij
  - First subscript is row, second is column

ID	Age	Gender	Income
Mary	a_11		
Bill			
John		a_32	
Larry			

Vectors

- Each row and each column in a matrix is a vector
- – Vertical vectors are column vectors, horizontal are row vectors
- Denoted by lowercase bold letter: **y**
- Each cell in the vector identified by subscript x_i

Ways and Modes

- Ways are the dimensions of a matrix.
- Modes are the sets of entities indexed by the ways of a matrix

	Event 1	Event 2	Event 3	Event 4
EVELYN	1	1	1	1
LAURA	1	1	1	0
THERESA	0	1	1	1
BRENDA	1	0	1	1
CHARLO	0	0	1	1
FRANCES	0	0	1	0
ELEANOR	0	0	0	0
PEARL	0	0	0	0
RUTH	0	0	0	0
VERNE	0	0	0	0
MYRNA	0	0	0	0
	0	0	0	0
	0	0	0	0

2-way, 2-mode

	Mary	Bill	John	Larry
Mary	0	1	0	1
Bill	1	0	0	1
John	0	1	0	0
Larry	1	0	1	0

2-way, 1-mode

Proximity Matrices

- Proximity Matrices record “degree of proximity”.
- Proximities are usually among a single set of actor (hence, they are 1-mode), but they are not limited to 1s and 0s in the data.
- What constitutes the *proximity* is user-defined.
    - Driving distances are one form of proximities, other forms might be number of friends in common, time spent together, number of emails exchanged, or a measure of similarity in cognitive structures.

Proximity Matrices

- Proximity matrices can contain either *similarity* or *distance* (or *dissimilarity*) data.
    - Similarity data, such as number of friends in common or correlations, means a larger number represents more similarity or greater proximity
    - Distance (or dissimilarity data) such as physical distance means a larger number represents more dissimilarity or less proximity

Transposes

- The transpose M' of a matrix M is the matrix flipped on its side.
  - The rows become columns and the columns become rows
  - So the transpose of an m by n matrix is an n by m matrix.

Transpose Example

M	Tennis	Football	Rugby	Golf
Mike	0	0	1	0
Ron	0	1	1	0
Pat	0	0	0	1
Bill	1	1	1	1
Joe	0	0	0	0
Rich	0	1	1	1
Peg	1	1	0	1

M^T	Mike	Ron	Pat	Bill	Joe	Rich	Peg
Tennis	0	0	0	1	0	0	1
Football	0	1	0	1	0	1	1
Rugby	1	1	0	1	0	1	0
Golf	0	0	1	1	0	1	1

Dichotomizing

- X is a valued matrix, say 1 to 10 rating of strength of tie
- Construct a matrix Y of ones and zeros s.t. $y_{ij} = 1$ if $x_{ij} > 5$, and $y_{ij} = 0$ otherwise

	EVE	**LAU**	**THE**	**BRE**	**CHA**
EVELYN	8	6	7	6	3
LAURA	6	7	6	6	3
THERESA	7	6	8	6	4
BRENDA	6	6	6	7	4
CHARLOTTE	3	3	4	4	4

	EVE	LAU	THE	BRE	CHA
EVELYN	1	1	1	1	0
LAURA	1	1	1	1	0
THERESA	1	1	1	1	0
BRENDA	1	1	1	1	0
CHARLOTTE	0	0	0	0	0

Symmetrizing

- When matrix is not symmetric, i.e., $x_{ij} \neq x_{ji}$
- Symmetrize various ways. Set y_{ij} and y_{ji} to:
  - Maximum(x_ij, x_ji): union rule;
  - Minimum(x_ij, x_ji): intersection rule;
  - Average (x_ij+x_ji)/2
  - Lowerhalf: choose x_{ij} when $i > j$ and x_{ji} otherwise

Symmetrizing Example

What rule are we using here?

	ROM	BON	AMB	BER	PET	LOU
ROMUL_10	0	1	1	0	3	0
BONAVEN_5	0	0	1	0	3	2
AMBROSE_9	0	1	0	0	0	0
BERTH_6	0	1	2	0	3	0
PETER_4	0	3	0	1	0	2
LOUIS_11	0	2	0	0	0	0

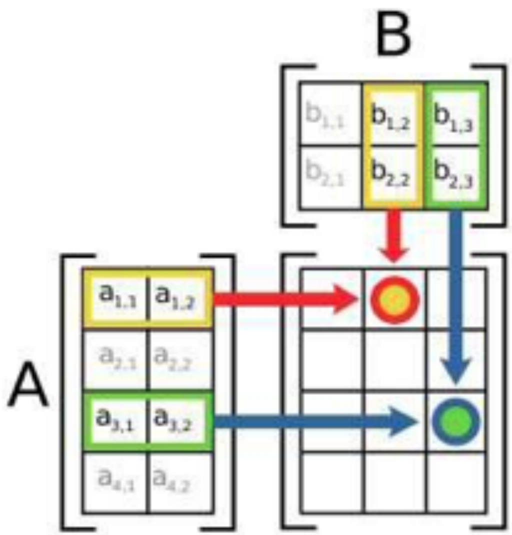
	ROM	BON	AMB	BER	PET	LOU
ROMUL_10	0	1	1	0	3	0
BONAVEN_5	1	0	1	1	3	2
AMBROSE_9	1	1	0	2	0	0
BERTH_6	0	1	2	0	3	0
PETER_4	3	3	0	3	0	2
LOUIS_11	0	2	0	0	2	0

Matrix Multiplication

- Matrix products are not generally commutative (i.e., AB does not usually equal BA)
- Notation: $C = AB$
- only possible when the number of columns in A equals number of rows in B; these are said to be comformable. It is calculated as:

$$\boldsymbol{c_{ij} = \sum a_{ik} * b_{kj} \quad \forall k}$$

Matrix multiplication example i



$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1\times 3+0\times 2+2\times 1 & 1\times 1+0\times 1+2\times 0 \\ -1\times 3+3\times 2+1\times 1 & -1\times 1+3\times 1+1\times 0 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$$

Assessing node's environment

X

	a	b	c	d	e	f
a	0	1	0	1	1	1
b	0	0	1	0	0	0
c	1	1	0	1	0	0
d	0	1	1	0	1	1
e	1	0	0	0	0	0
f	1	1	0	0	1	0

A

	hrs	$	lib
a	3	50	1
b	9	10	4
c	3	5	3
d	7	30	5
e	1	20	2
f	5	5	4

XA

	hrs	$	lib
a	22	65	15
b	3	5	3
c	19	90	10
d	18	40	13
e	3	50	1
f	13	80	7

- Hrs and $ columns of XA give social access to resources
- Lib column gives how liberal the person's social environment is

Boolean matrix multiplication

- Values can be 0 or 1 for all matrices
- Products are dichotomized

Would have been a 2 in regular matrix multiplication

	Mary	Bill	John	Larry
Mary	0	1	0	1
Bill	1	0	1	0
John	0	0	0	1
Larry	0	0	0	0

A

	Mary	Bill	John	Larry
Mary	0	0	1	1
Bill	1	0	1	0
John	0	0	0	1
Larry	0	1	0	0

B

	Mary	Bill	John	Larry
Mary	1	1	1	0
Bill	0	0	0	1
John	0	1	0	0
Larry	0	0	0	0

AB

Composition of relations

- We represent each social relation (e.g., F= friend of, B = boss of) as a matrix
- To create the compound relation friend of the boss of (FB), we just multiply the two matrices

F

	a	b	c	d
a	0	1	0	1
b	1	0	1	0
c	1	1	0	1
d	1	0	1	0

x

B

	a	b	c	d
a	0	0	1	1
b	1	0	0	0
c	0	1	0	0
d	0	0	0	0

=

FB

	a	b	c	d
a	1	0	0	0
b	0	1	1	1
c	1	0	1	1
d	0	1	1	1

Composition of relations

Hard for *a* to borrow any subordinates

F

	a	b	c	d
a	0	1	0	1
b	1	0	1	0
c	1	1	0	1
d	1	0	1	0

x

B

	a	b	c	d
a	0	0	1	1
b	1	0	0	0
c	0	1	0	0
d	0	0	0	0

=

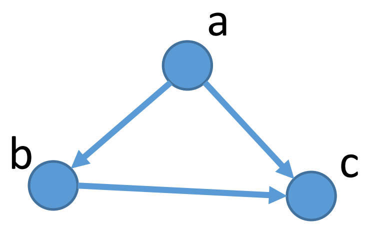
FB

	a	b	c	d
a	1	0	0	0
b	0	1	1	1
c	1	0	1	1
d	0	1	1	1

Everyone is friends with their boss

- FB(c,a) = 1 (or cFBa) means that person c is friend of someone (namely b) who is the boss of a. i.e., c is friends with a’s boss
- FB(a,a) = 1 (or aFBa) means person a is friends with someone (b again) who is a’s boss. i.e., *a* is friends with her boss
- FB(b,d) = 1, so person b is friends with someone (a) who is the boss of d

Transitivity



- L = "likes someone", uLLv means u likes someone who likes v

L

	a	b	c	d
a	0	1	0	1
b	0	0	1	0
c	0	1	0	0
d	1	0	1	0

x

L

	a	b	c	d
a	0	1	0	1
b	0	0	1	0
c	0	1	0	0
d	1	0	1	0

=

LL

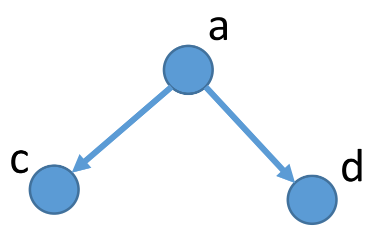
	a	b	c	d
a	1	0	2	0
b	0	1	0	0
c	0	0	1	0
d	0	2	0	1

Note diagonal of LL is all 1s, so everyone is lucky enough to like someone who likes them

LL(d,b) = 2 indicates d likes 2 people who like b

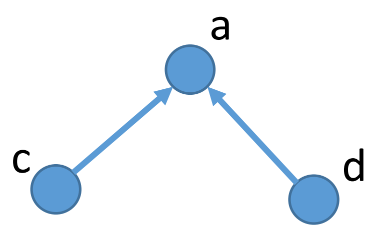
- If *a* likes b and b likes c, does that mean *a* likes c?
- If matrix L = matrix LL, then L is a transitive relation, in keeping with balance theory

Converse of a relation



“boss of”

- In relational terms, the converse of a relation is the reciprocal role
  - Converse of “boss of” is “subordinate of”
- In graph terms, we are just reversing the direction of arrows
- In matrix terms, we are transposing matrix
  - Construct B’ (reports to) from B (is the boss of)



“reports to”

B

	a	b	c	d
a	0	0	1	1
b	1	0	0	0
c	0	1	0	0
d	0	0	0	0

Boss of

B'

	a	b	c	d
a	0	1	0	0
b	0	0	1	0
c	1	0	0	0
d	1	0	0	0

Reports to

To transpose a matrix, write each row as a column

Composition of relations – with converse

- To create the compound relation friend of the subordinate of (FB’), we just post-multiply F by the transpose of B
- FB’(c,a) = 1 (or cFB’a) means that person c is friend of someone (namely *d*) who is a subordinate of *a*. i.e., c is friends with *a*’s subordinate
- FB’(a,a) = 1 (or aFB’a) means person *a* is friends with someone (*d*) who is her subordinate. i.e., *a* is friends with one of her direct reports.

F

	a	b	c	d
a	0	1	0	1
b	1	0	1	0
c	1	1	0	1
d	1	0	1	0

x

B'

	a	b	c	d
a	0	1	0	0
b	0	0	1	0
c	1	0	0	0
d	1	0	0	0

=

FB’

	a	b	c	d
a	1	0	1	0
b	1	1	0	0
c	1	1	1	0
d	1	1	0	0

Everybody likes a’s subordinates

Products of matrices & their transposes

- XX' = product of matrix X by its transpose

$$(XX')_{ij} = \sum_k x_{ik} x_{jk}$$

- Computes sums of products of each pair of rows (cross-products)
- Similarities among rows

	1	2	3	4
Mary	0	1	1	1
Bill	1	0	1	0
John	0	0	0	1
Larry	0	0	0	0
Tina	1	1	1	0

X

	Mary	Bill	John	Larry	Tina
1	0	1	0	0	1
2	1	0	0	0	1
3	1	1	0	0	0
4	1	0	1	0	0

X'

	Mary	Bill	John	Larry	Tina
Mary	3	1	1	0	1
Bill	1	2	0	0	1
John	1	0	1	0	0
Larry	0	0	0	0	0
Tina	2	2	0	0	2

XX'

Multiplying a matrix by its transpose

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14
EVELYN	1	1	1	1	1	1	0	1	1	0	0	0	0	0
LAURA	1	1	1	0	1	1	1	1	0	0	0	0	0	0
THERESA	0	1	1	1	1	1	1	1	1	0	0	0	0	0
BRENDA	1	0	1	1	1	1	1	1	0	0	0	0	0	0
CHARLOTTE	0	0	1	1	1	0	1	0	0	0	0	0	0	0
FRANCES	0	0	1	0	1	1	0	1	0	0	0	0	0	0
ELEANOR	0	0	0	0	1	1	1	1	0	0	0	0	0	0
PEARL	0	0	0	0	0	1	0	1	1	0	0	0	0	0
RUTH	0	0	0	0	1	0	1	1	1	0	0	0	0	0
VERNE	0	0	0	0	0	0	1	1	1	0	0	1	0	0
MYRNA	0	0	0	0	0	0	0	1	1	1	0	1	0	0
KATHERINE	0	0	0	0	0	0	0	1	1	1	0	1	1	1
SYLVIA	0	0	0	0	0	0	1	1	1	1	0	1	1	1
NORA	0	0	0	0	0	1	1	0	1	1	1	1	1	1
HELEN	0	0	0	0	0	0	1	1	0	1	1	1	0	0
DOROTHY	0	0	0	0	0	0	0	1	1	0	0	0	0	0
OLIVIA	0	0	0	0	0	0	0	0	1	0	1	0	0	0
FLORA	0	0	0	0	0	0	0	0	1	0	1	0	0	0

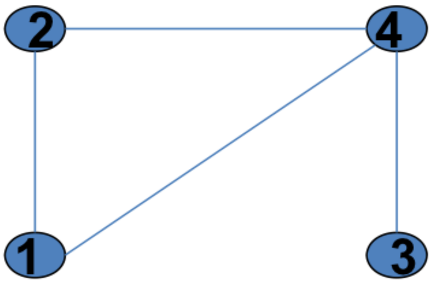
	EV	LA	TH	BR	CH	FR	EL	PE	RU	VE	MY	KA	SY	NO	HE	DO	OL	FL
E1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E3	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
E4	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
E5	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0
E6	1	1	1	1	0	1	1	1	0	0	0	0	0	1	0	0	0	0
E7	0	1	1	1	1	0	1	0	1	1	0	0	1	1	1	0	0	0
E8	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	0	0
E9	1	0	1	0	0	0	0	1	1	1	1	1	1	1	0	1	1	1
E10	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
E11	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1
E12	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0
E13	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
E14	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0

	EVE	LAU	THE	BRE	CHA	FRA	ELE	PEA	RUT	VER	MYR	KAT	SYL	NOR	HEL	DOR	OLI	FLO
EVELYN	8	6	7	6	3	4	3	3	3	2	2	2	2	2	1	2	1	1
LAURA	6	7	6	6	3	4	4	2	3	2	1	1	2	2	2	1	0	0
THERESA	7	6	8	6	4	4	4	3	4	3	2	2	3	3	2	2	1	1
BRENDA	6	6	6	7	4	4	4	2	3	2	1	1	2	2	2	1	0	0
CHARLOTTE	3	3	4	4	4	2	2	0	2	1	0	0	1	1	1	0	0	0
FRANCES	4	4	4	4	2	4	3	2	2	1	1	1	1	1	1	1	0	0
ELEANOR	3	4	4	4	2	3	4	2	3	2	1	1	2	2	2	1	0	0
PEARL	3	2	3	2	0	2	2	3	2	2	2	2	2	2	1	2	1	1
RUTH	3	3	4	3	2	2	3	2	4	3	2	2	3	2	2	2	1	1
VERNE	2	2	3	2	1	1	2	2	3	4	3	3	4	3	3	2	1	1
MYRNA	2	1	2	1	0	1	1	2	2	3	4	4	4	3	3	2	1	1
KATHERINE	2	1	2	1	0	1	1	2	2	3	4	6	6	5	3	2	1	1
SYLVIA	2	2	3	2	1	1	2	2	3	4	4	6	7	6	4	2	1	1
NORA	2	2	3	2	1	1	2	2	2	3	3	5	6	8	4	1	2	2
HELEN	1	2	2	2	1	1	2	1	2	3	3	3	4	4	5	1	1	1
DOROTHY	2	1	2	1	0	1	1	2	2	2	2	2	2	1	1	2	1	1
OLIVIA	1	0	1	0	0	0	0	1	1	1	1	1	1	2	1	1	2	2
FLORA	1	0	1	0	0	0	0	1	1	1	1	1	1	2	1	1	2	2

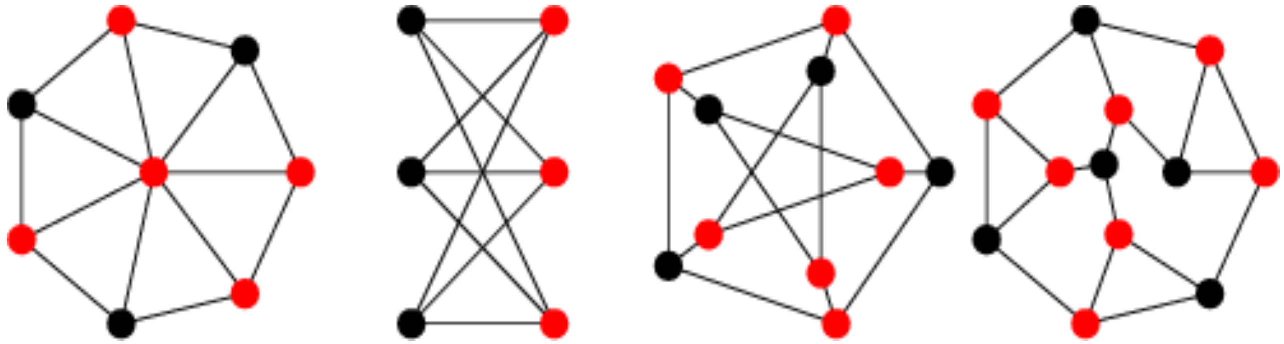
squaring an adjacency matrix

$$g = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$g^2 = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 3 \end{pmatrix}$$



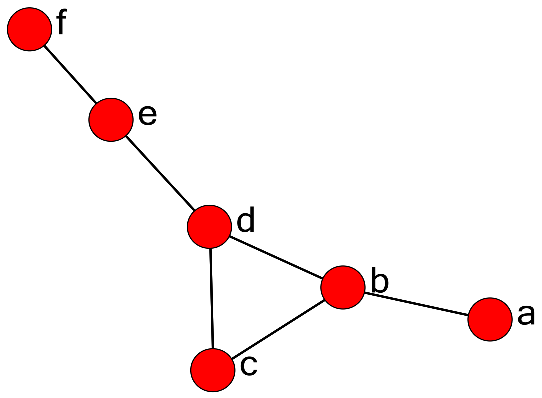
number of walks of length 2 from i to j



Graph Theoretic Concepts

Intro to graph terminology

- Nodes
  - Aka vertices or points in more mathematical work
  - Actors, agents, egos, alters, contacts in more sociological work
  - Nodes can individuals or collective actors, such as countries
  - In social network analysis, nodes typically have agency
- Ties
  - Aka edges, arcs or lines in more technical work
  - Links, bonds, direct connections etc in more sociological work
  - Ties are typically binary: they link exactly two nodes

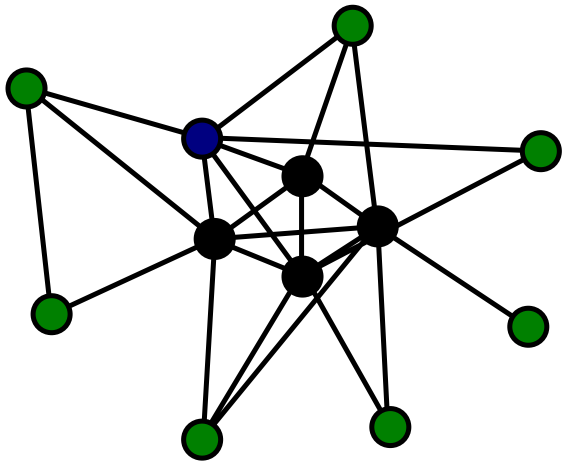


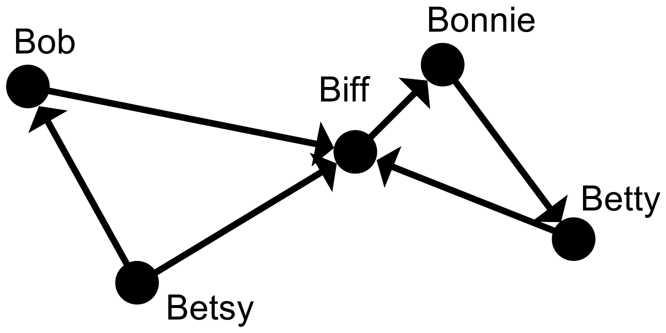
A graph

- G(V,E) is …
- A set of vertices V, together with …
- A set of edges E
- The edges are binary, meaning they have exactly two endpoints
    - They are 2-tuples
- If the edges are k-tuples (where $k > 2$), they comprise a hyper-graph

Directed and undirected graphs

- Graphs can be directed or undirected*
- Undirected
  - In an undirected graph, the ties don't have direction – two nodes u and v are connected by a tie, but it doesn't matter whether you say u has tie to v or v has tie to u.
    - E.g., married, taking same class, siblings
  - The ties are called edges
- Directed
  - Ties (which are called arcs) have direction. If u has a tie to v, it may or may not be true that v has a tie to u
    - Gives advice to; sends an email to; thinks well of
  - Directed graphs often called digraphs
- An undirected graph is like a directed graph in which all arcs are reciprocated, but technically there is a difference
  - In an undirected graph, non-reciprocity is impossible/insensible





*But in some usages graph refers to both, m to the species as a whole, while other times it contrasts with 'woman'

Transpose Adjacency matrix

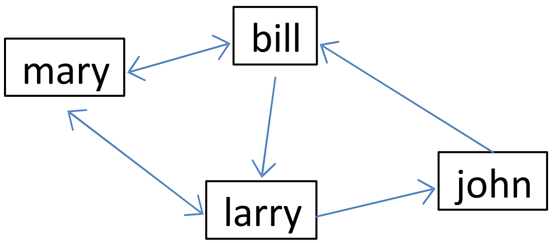
- In directed graphs, interchanging rows/columns of adjacency matrix effectively reverses the direction & meaning of ties

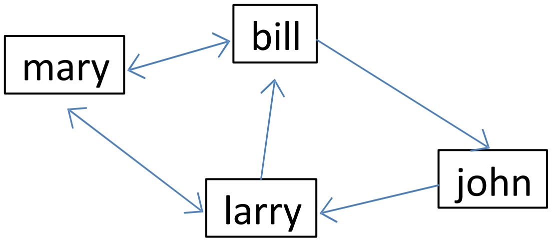
	Mary	Bill	John	Larry
Mary	0	1	0	1
Bill	1	0	0	1
John	0	1	0	0
Larry	1	0	1	0

Gives money to

	Mary	Bill	John	Larry
Mary	0	1	0	1
Bill	1	0	1	0
John	0	0	0	1
Larry	1	1	0	0

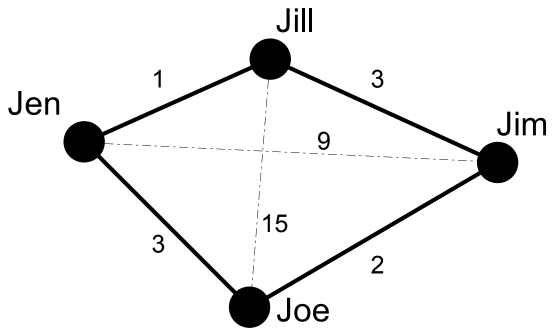
Gets money from

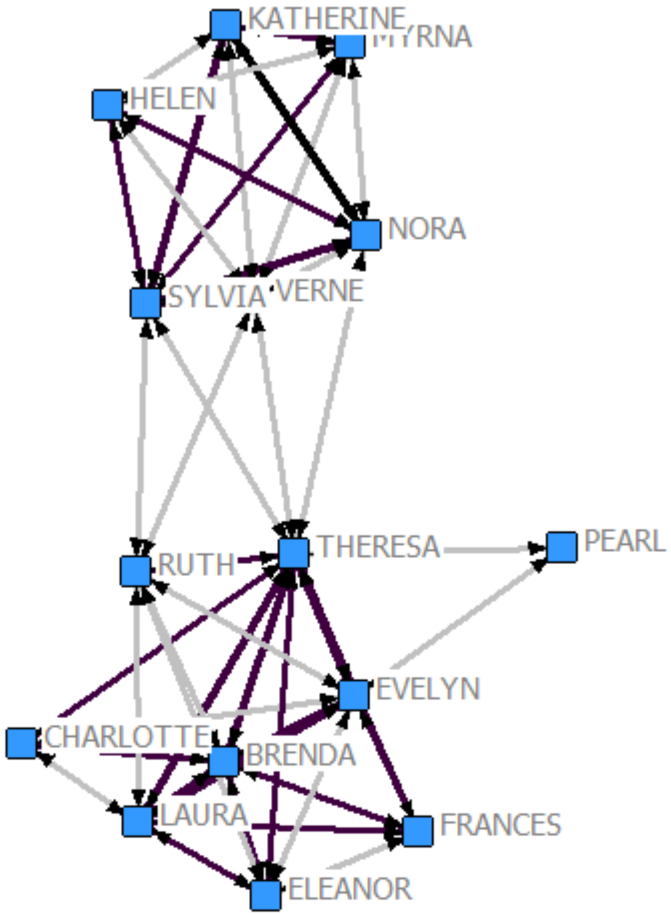




Valued networks

- We can attach values to ties, representing quantitative properties of the relationship
- G(V,E,F), where F is a function delivering real values
    - Strength of relationship
    - Information capacity of tie
    - Rates of flow or traffic across tie
    - Distances between nodes
    - Probabilities of passing on information
    - Frequency of interaction





Valued Adjacency Matrix

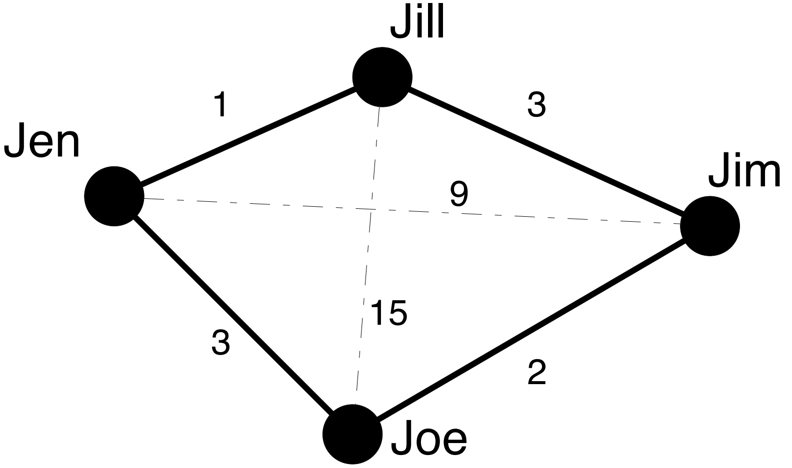
Dichotomized

	Jim	Jill	Jen	Joe
Jim	-	1	0	1
Jill	1	-	1	0
Jen	0	1	-	1
Joe	1	0	1	-

Distances btw offices

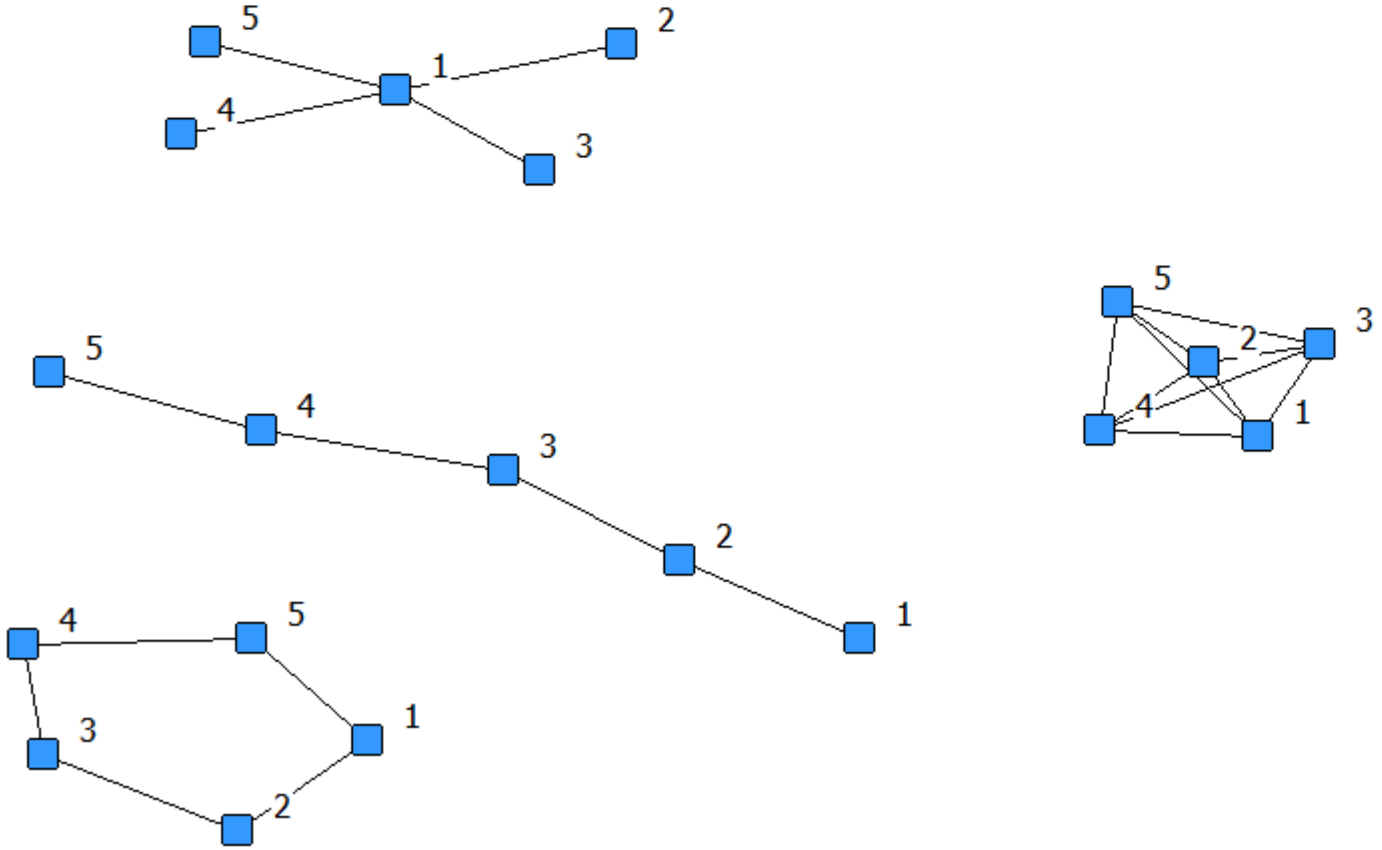
	Jim	Jill	Jen	Joe
Jim	-	3	9	2
Jill	3	-	1	15
Jen	9	1	-	3
Joe	2	15	3	-

- The diagram below uses solid lines to represent the adjacency matrix, while the numbers along the solid line (and dotted lines where necessary) represent the proximity matrix.
- In this particular case, one can derive the adjacency matrix by dichotomizing the proximity matrix on a condition of $p_{ij} <= 3$.



Some well-known graphs

- Line/path
- Circle/cycle
- Clique
- Star



Expressing the presence of a tie

- Suppose you have an undirected graph G(V,E)
  - To express that u and v have a tie in this graph we can write (u,v) ϵ E or, if there multiple graphs under discussion, (u,v) ϵ E(G)
  - It is irrelevant whether we write (u,v) ϵ E or (v,u) ϵ E
- If G(V,E) is directed, then
  - (u,v) ϵ E means u has a tie to v.
  - If it also true that (v,u) ϵ E , we say the u--v tie is reciprocated

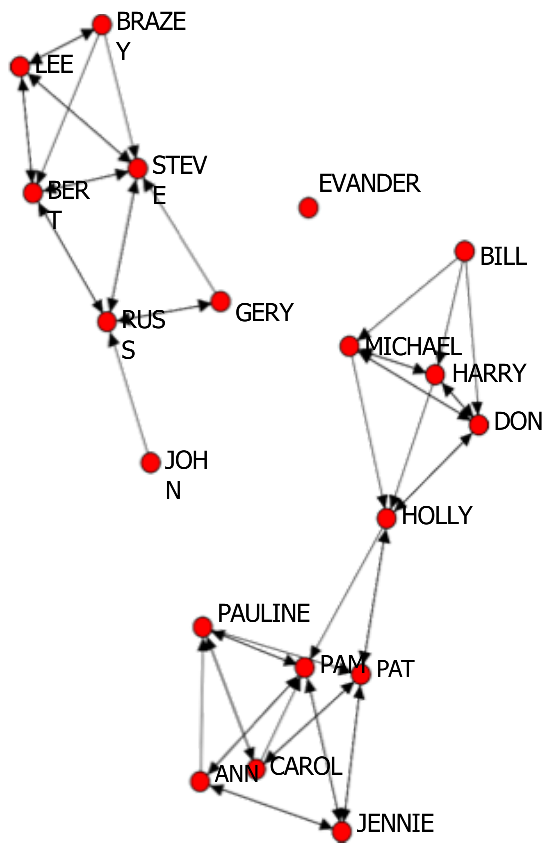
Relational terminology

- Suppose B is the relation “is the brother of” and F is the relation “is the father of”
  - uBv means u is the brother of v
  - yFx means y is the father of x
- We can define a compound relation BF as “is the brother of someone who is the father of”
  - uBFx means u is the brother of the father of x
- So BF is the uncle relation
  - U = BF
  - zUx means z is the uncle of x

Relational terminology – cont.

- The relation FF is the father of the father of
  - uFFv means that u is the grandfather of v
- We use F‘ to indicate the converse of a relation F
- If F means is the father of, then F’ means is the child of
  - uFv if and only if vF’U
- The compound relation F’F means ‘the child of the father of’
  - uF’Fv means that u is the child of someone who is the father of v.
  - Who are u and v to each other? They are siblings
- The relation FF’ is the father of the child of
  - uFF’v means that u is the father of someone who is the son of v
  - In other words u and v are co-parents to each other – they have the same children

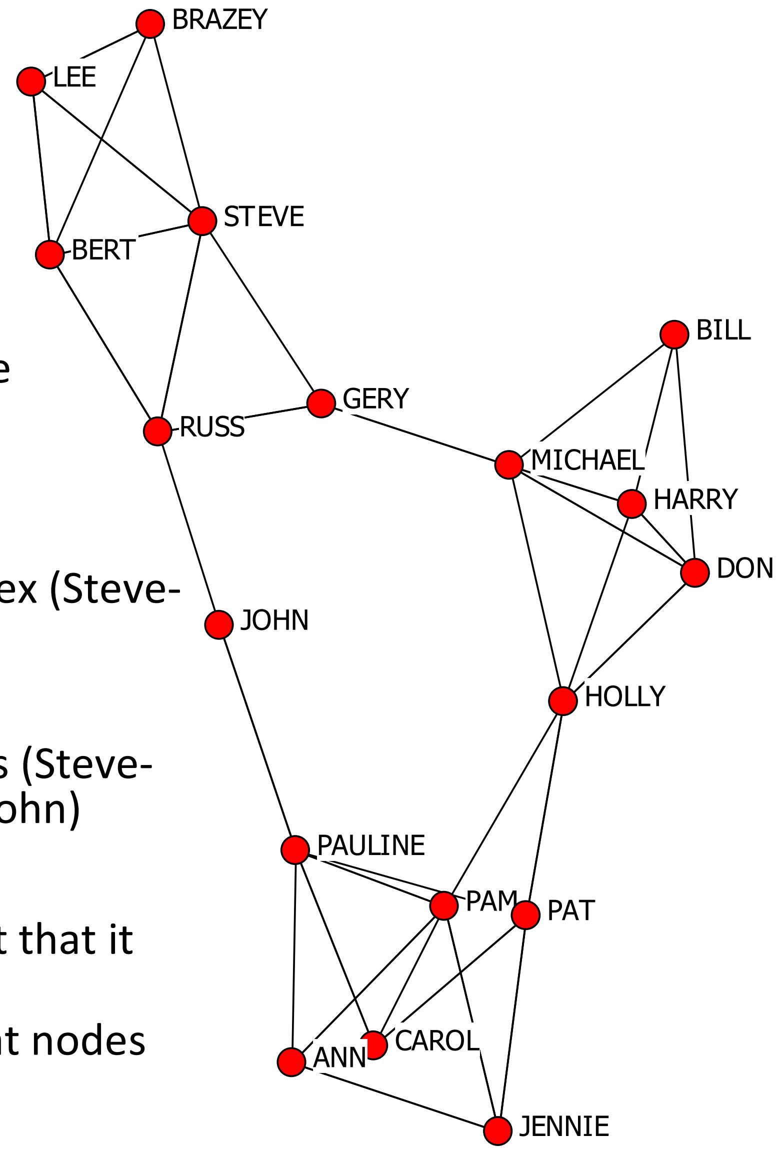
Node-related concepts



- **Degree**
  - The number of ties incident upon a node
  - In a digraph, we have indegree (number of arcs to a node) and outdegree (number of arcs from a node)
- **Pendant**
  - A node connected to a component through only one edge or arc
    - A node with degree 1
    - Example: John
- **Isolate**
  - A node which is a component on its own
    - E.g., Evander

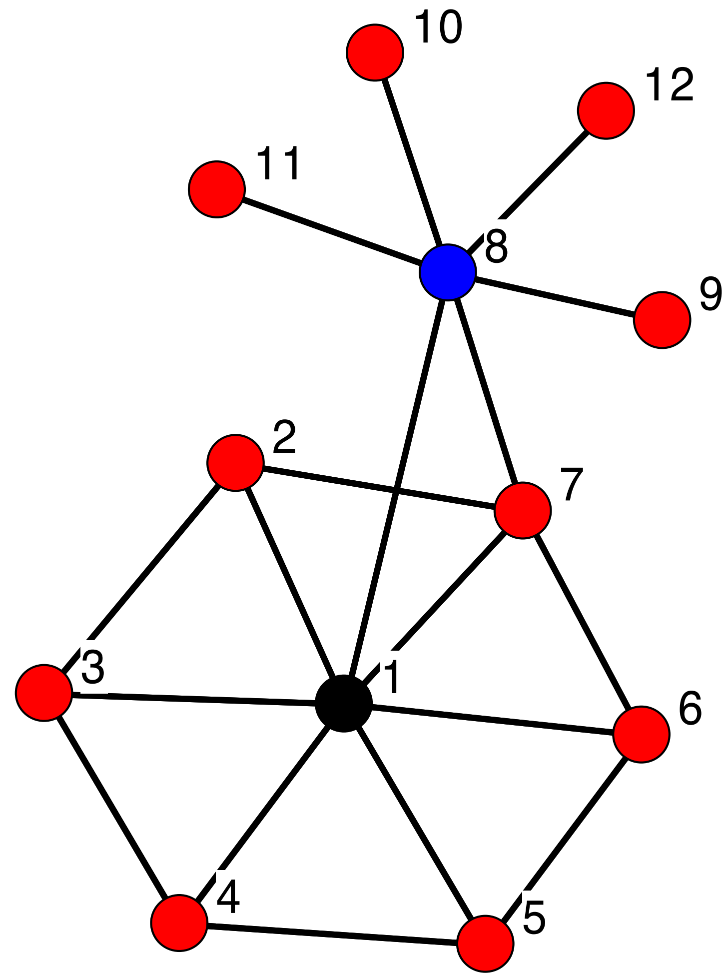
Graph traversals

- Walk
  – Any unrestricted traversing of vertices across edges (Russ-Steve-Bert-Lee-Steve)
- Trail
  – A walk restricted by not repeating an edge or arc, although vertices can be revisited (Steve-Bert-Lee-Steve-Russ)
- Path
  – A trail restricted by not revisiting any vertex (Steve-Lee-Bert-Russ)
- Geodesic Path
  – The shortest path(s) between two vertices (Steve-Russ-John is shortest path from Steve to John)
- Cycle
  – A cycle is in all ways just like a path except that it ends where it begins
  – Aside from endpoints, cycles do not repeat nodes
  – E.g. Brazey-Lee-Bert-Steve-Brazey



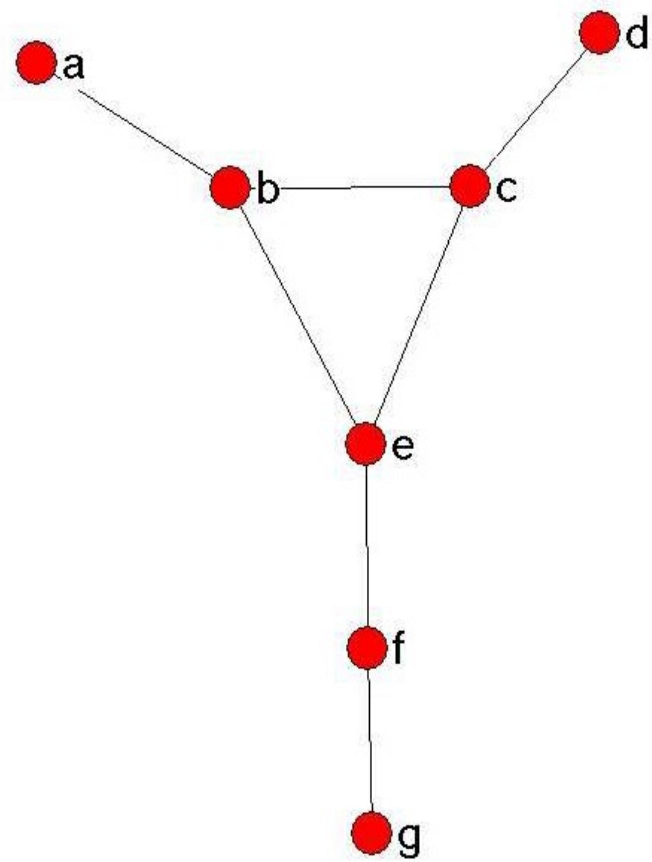
Length & Distance

- Length of a path (or any walk) is the number of links it has
- The **Geodesic Distance** (aka graph-theoretic distance) between two nodes is the length of the shortest path
  - Distance from 5 to 8 is 2, because the shortest path (5-1-8) has two links



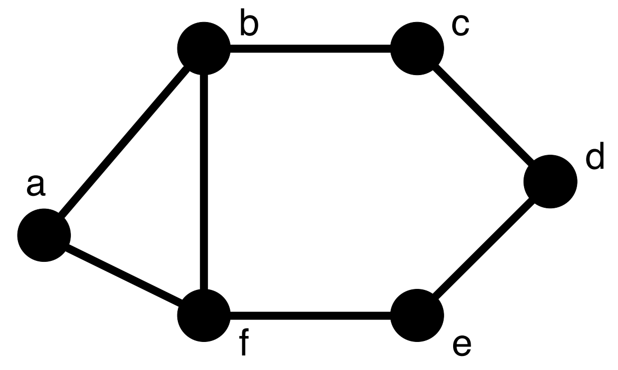
Geodesic Distance Matrix

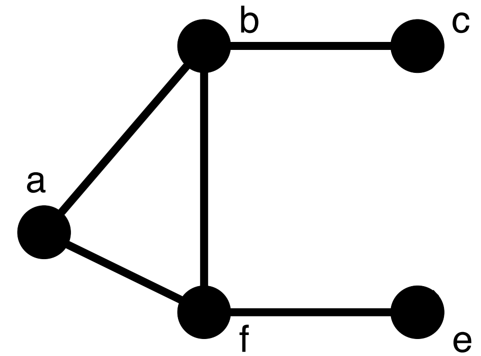
	a	b	c	d	e	f	g
a	0	1	2	3	2	3	4
b	1	0	1	2	1	2	3
c	2	1	0	1	1	2	3
d	3	2	1	0	2	3	4
e	2	1	1	2	0	1	2
f	3	2	2	3	1	0	1
g	4	3	3	4	2	1	0



Subgraphs

- Set of nodes
  - Is just a set of nodes
- A subgraph
  - Is set of nodes together with ties among them
- An induced subgraph
  - Subgraph defined by a set of nodes
  - Like pulling the nodes and ties out of the original graph

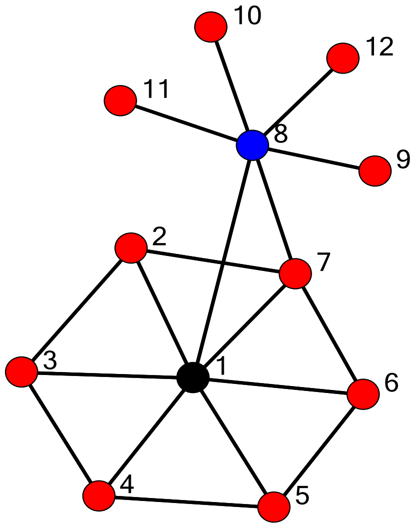




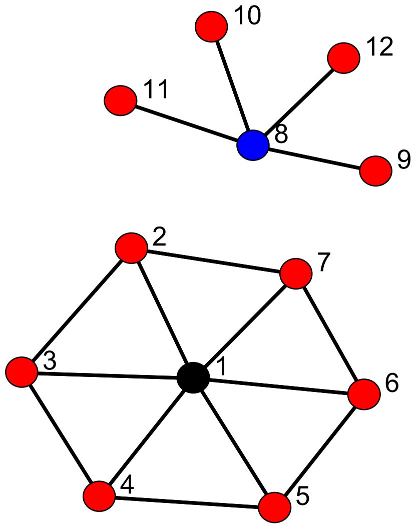
Subgraph induced by considering the set {a,b,c,f,e}

Connected vs disconnected graphs

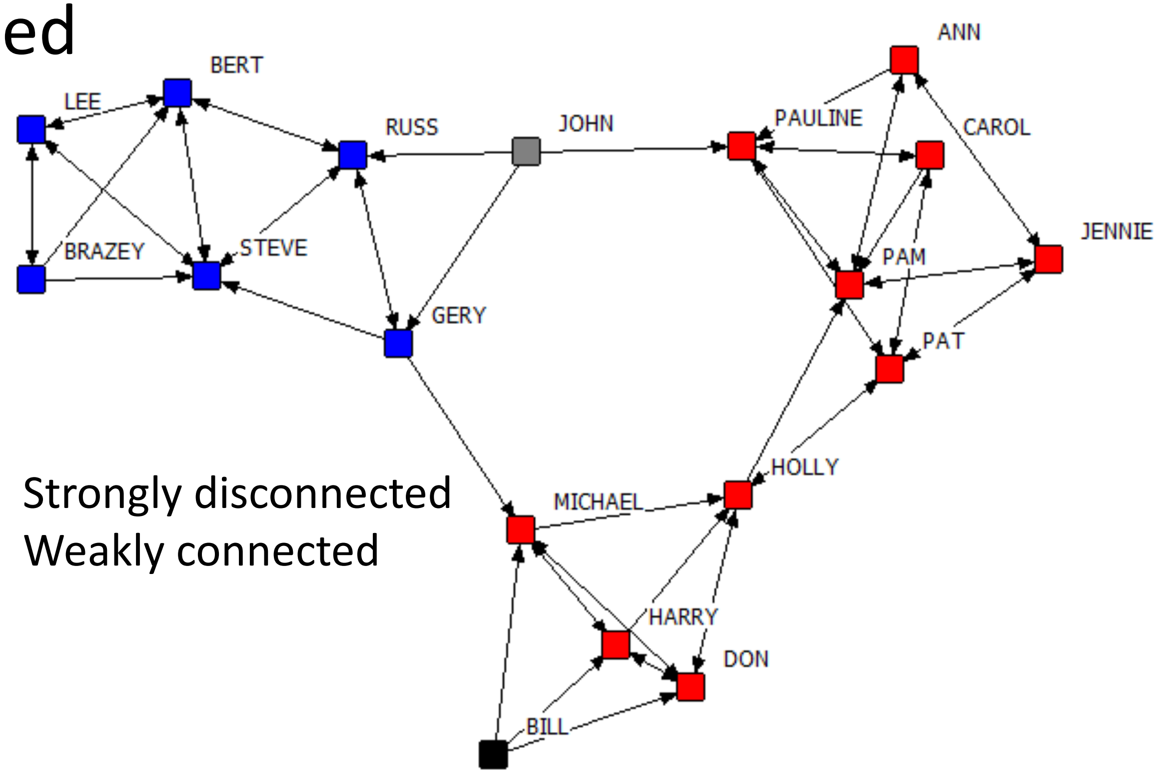
- A graph is connected if you can reach any node from any other – i.e., there exists a path from one to the other
- Directed graphs are often disconnected



Connected



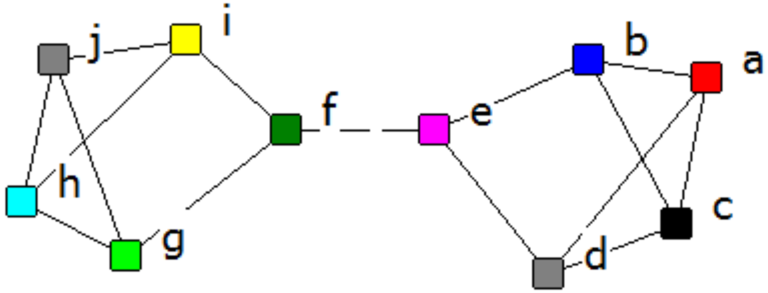
Disconnected



Strongly disconnected
Weakly connected

Component

- Maximal sets of nodes in which every node can reach every other by some path (no matter how long)
  - Coherent fragments of a graph
- A graph with a single component is called a connected graph
- Weak vs strong components
  - A weak component is where we ignore the direction of the arcs



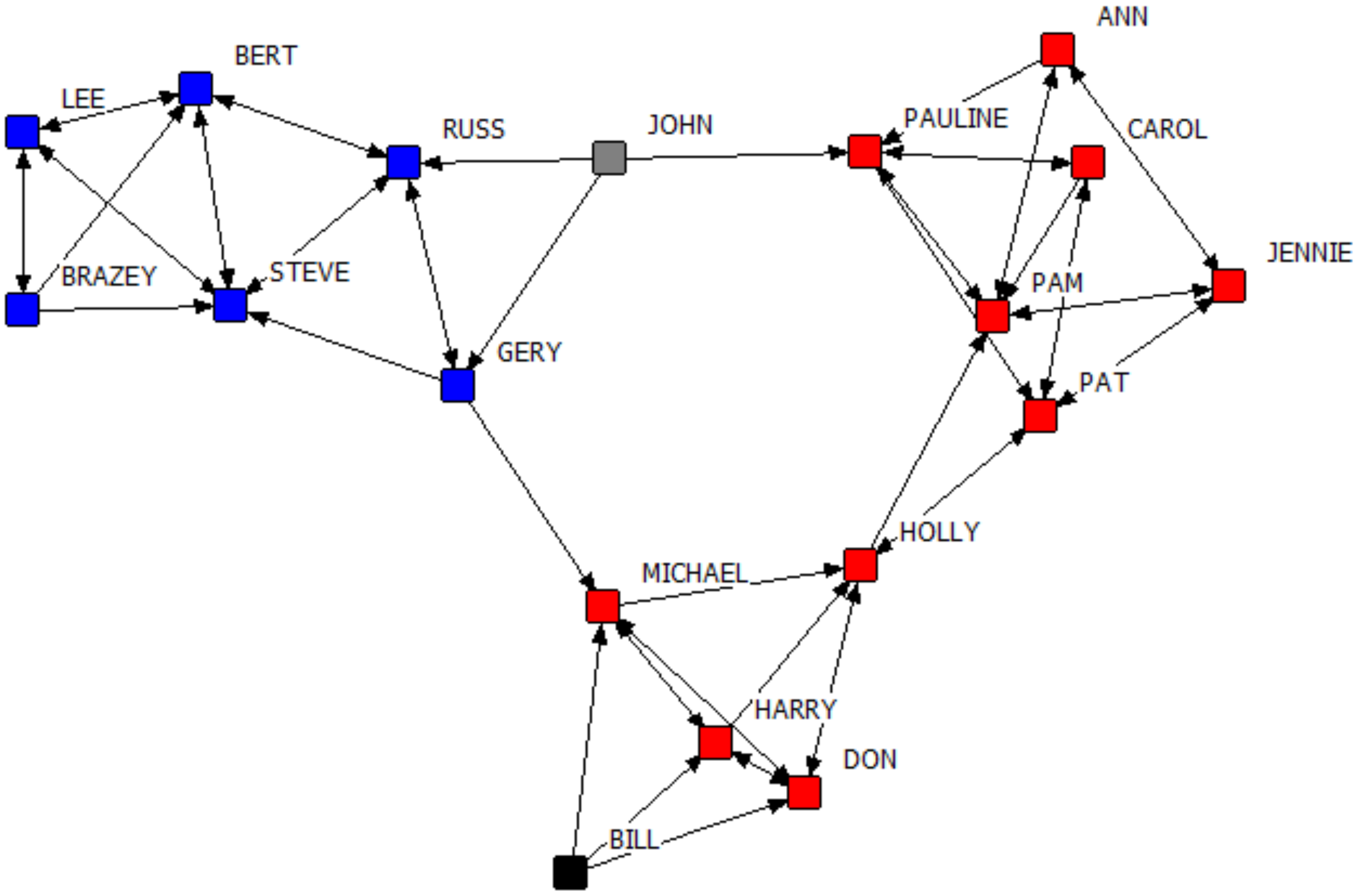
Removing F-E tie would create a network with 2 components

It is relations (types of tie) that define different networks, not components. A network that has two components remains one (disconnected) network.

Components in Directed Graphs

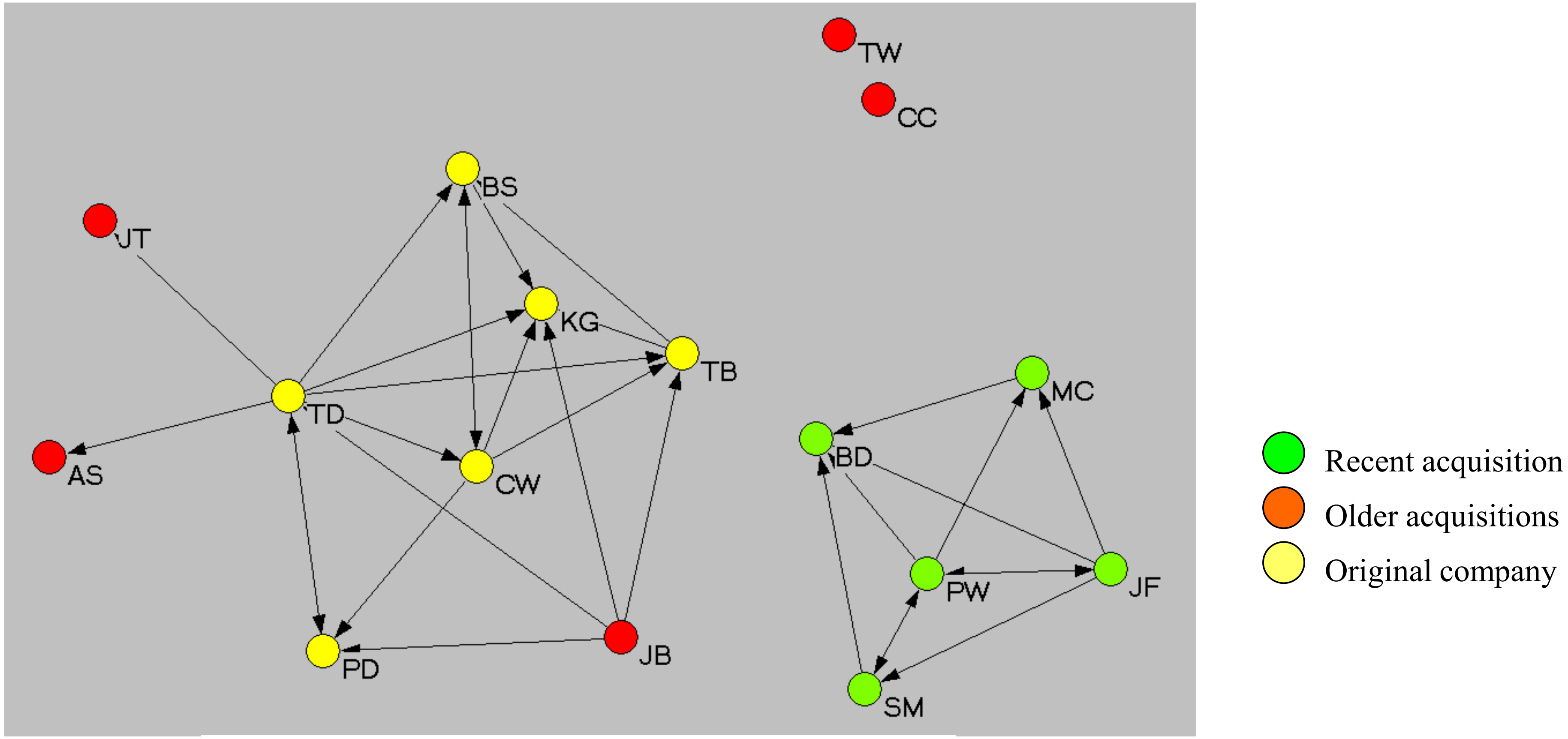
- Strong component
  - There is a directed path from each member of the component to every other
- Weak component
  - There is an undirected path (a weak path) from every member of the component to every other
  - Is like ignoring the direction of ties – driving the wrong way if you have to

1 weak component, 4 strong components



A network with 4 weak components

Who you go to so that you can say 'I ran it by ____, and she says ...'

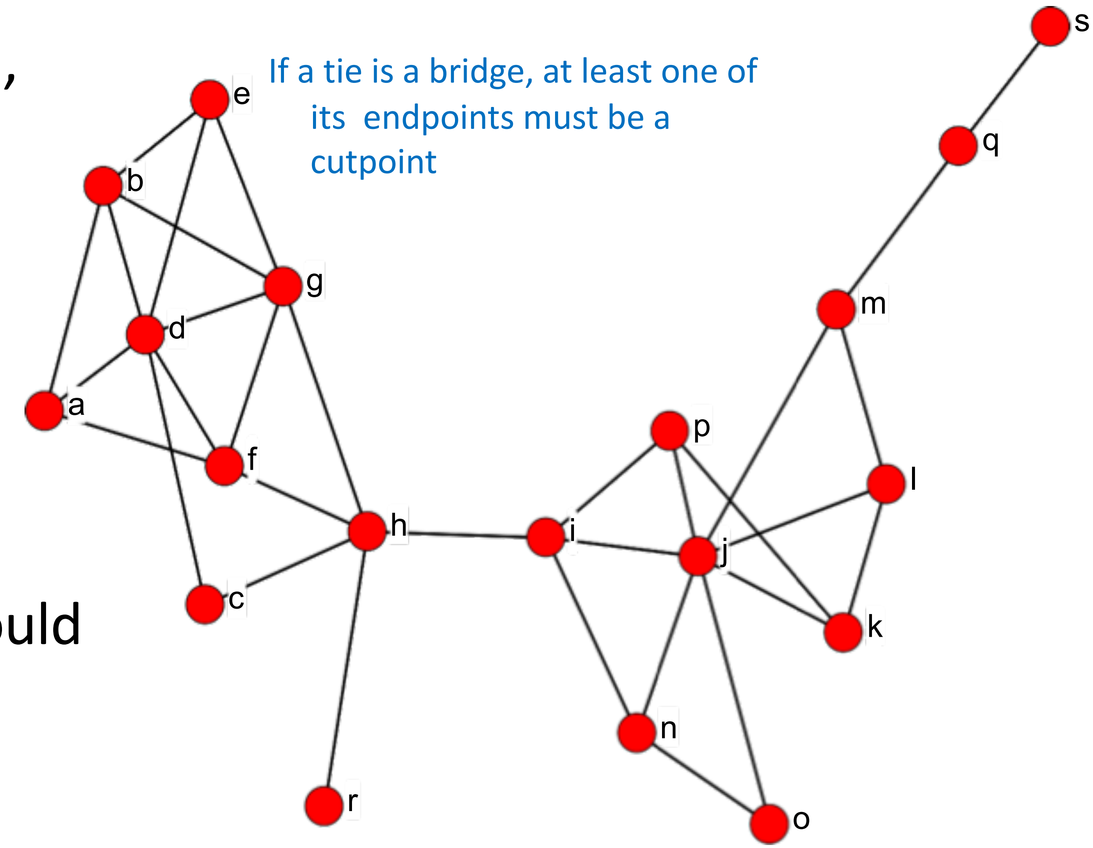


Data drawn from Cross, Borgatti & Parker 2001.

Cutpoints and Bridges

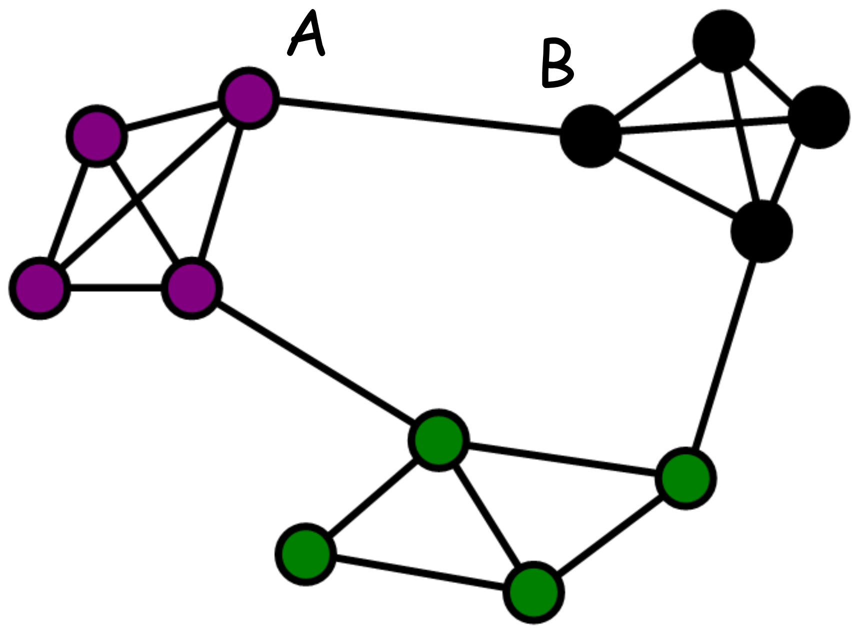
- Cutpoint
  - A node which, if deleted, would increase the number of components
- Bridge
  - A tie that, if removed, would increase the number of components

If a tie is a bridge, at least one of its endpoints must be a cutpoint



Local Bridge of Degree K

- An edge that connects nodes that would otherwise be a minimum of *k* steps apart
  - The A—B tie is local bridge of degree 5
- Loss of relationship between A and B would effectively, though not actually, disconnect A from B



Getting the Data in UCINET

- Four options:
  - DL Files
    - Text files of various formats that can be created easily by geeks and nerds
  - Excel Files/Grid format
    - UCINET has a spreadsheet tool that easily interacts with Excel or can allow manual entry if network is not too large
  - VNA Files
    - Text files that allow for a single-file that contains both dyadic and nodal attribute data
  - Import Text Via Spreadsheet tool
    - A new tool in UCINET that lets you do DL file formats in a spreadsheet tool

DL Files

- These are the most versatile
- There are multiple formats:
  - Full Matrix
  - Nodelist
  - Edgelist
- Each has its advantages

DL Data Formats

<table>
<tr>
<td>Dl n=5
Format = edgelist
Labels embedded
Data:
billy john 6
john billy 1
john jill 2
jill mary
mary billy 5
mary jill
mary jill

Best for data coming from a relational databases or if you have valued data.

Values are added if repeated and default to 1</td>
<td>Dl n=5
Format = nodelist
Labels embedded
Data:
billy john
john billy jill
jill mary
mary billy jill

This method is best for BINARY data

NOTE: This is a dichotomized version of the others</td>
<td>Dl n=5
Format = edgelist2
Labels embedded
Data:
billy Essex 4
john Cambridge 2
jill Oxford 3
mary Leeds 6

This is the same as the edgelist format, except the nominating node (the first column) is of a different MODE than the nominated node (the second column).

There is also nodelist2</td>
</tr>
</table>

VNA Files

- These **CAN** combine in one file both:
  - Nodal (attribute) data **and**
    - e.g., Age, gender, Education Level
  - Network/Relational/Dyadic data
    - E.g., Communicates with, Trusts
- Can have textual data
  - NetDraw will preserve the labels
  - UCINET will transform them to numbers

Sample VNA File

*Node data
"ID", "Gender", "Role"
 "HOLLY" “FEMALE" “STUDENT"
“STEVE" “MALE" “TEACHER"
 "CAROL" “FEMALE" “STUDENT"
…
*Tie data
FROM TO "campnet"
"HOLLY" "PAM" 1
"HOLLY" "PAT" 1
"BRAZEY" "STEVE" 1
"BRAZEY" "BERT" 1
"CAROL" "PAM" 1
"PAM" "ANN" 1
"PAT" "HOLLY" 1

Excel/Data Grid

- Excel is the “Universal Translator”
- UCINET has a Data Grid tool that
  - Looks like excel
  - Reads excel files
  - Works really well with Excel Cut&Paste
    - As long as you click in the right place for pasting your data

Some tricks

- If the network is small (not too many people)
  - I use excel
  - Create a comma-separated full-matrix-style file and cut and paste into the data grid
  - Manually create attribute file in UCINET (#s only)
- If the network is larger
  - I create an edgelist DL file for the network only
  - And a VNA file just with node data (attributes)
  - Then I:
    - Import the DL file into UCINET (creating ##h & ##d files)
    - Open the vna file as an attribute file
    - If I want to do attribute-based analyses in UCINET, I export the Attributes as a UCINET dataset (will translate text to numbers automatically for me- but I can't control them)

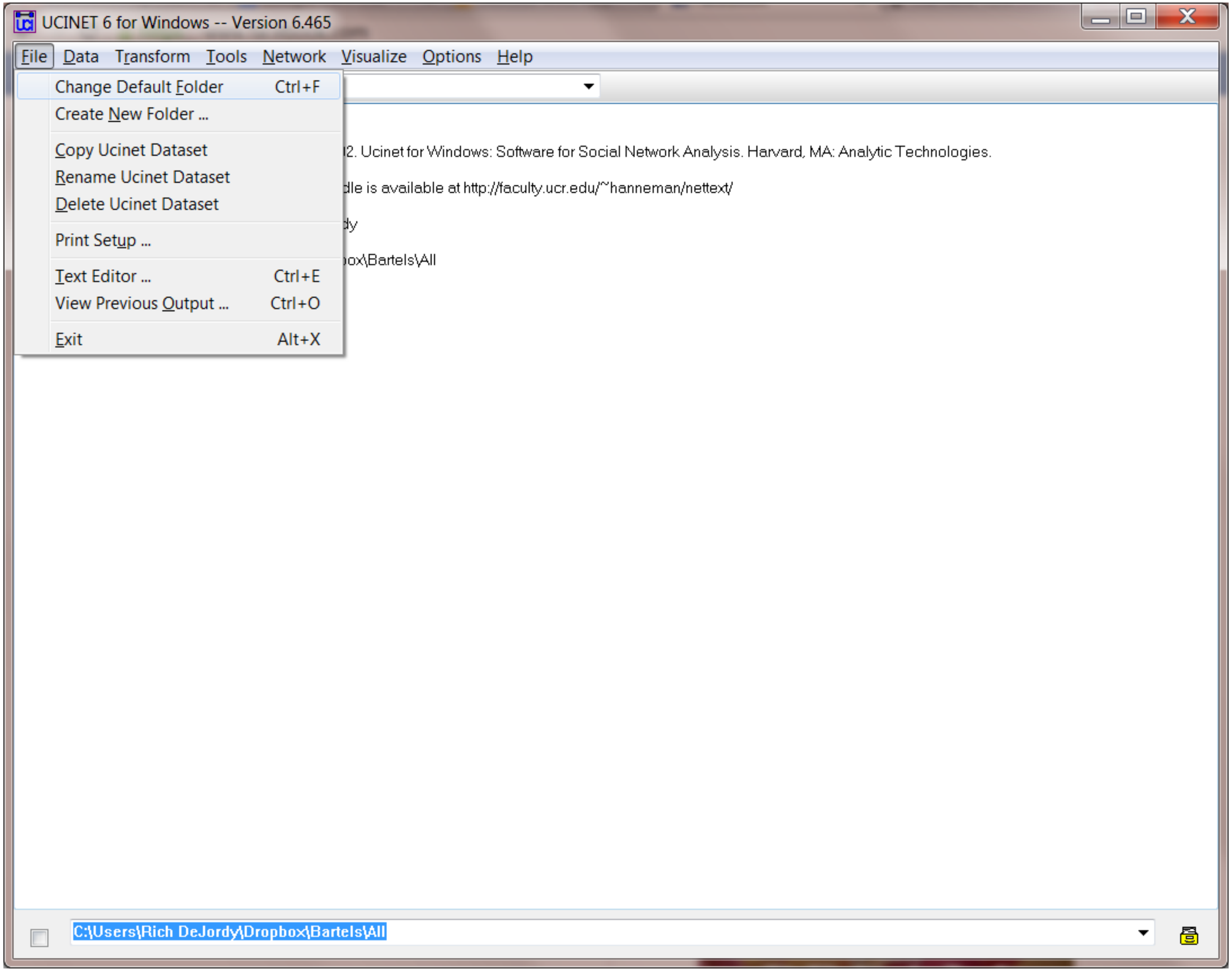
Where to find the importing

- In UCINET
  - Data | import | DL
  - Data | Import | VNA
  - Data | Spreadsheets | Matrix (Ctrl-S)
  - Data | Import via Spreasheet | DL
- In NetDraw
  - File | Open | Ucinet DL Text file
  - File | Open | VNA text file
  - NetDraw can work with the text files (no UCINET dataset). UCINET does not.

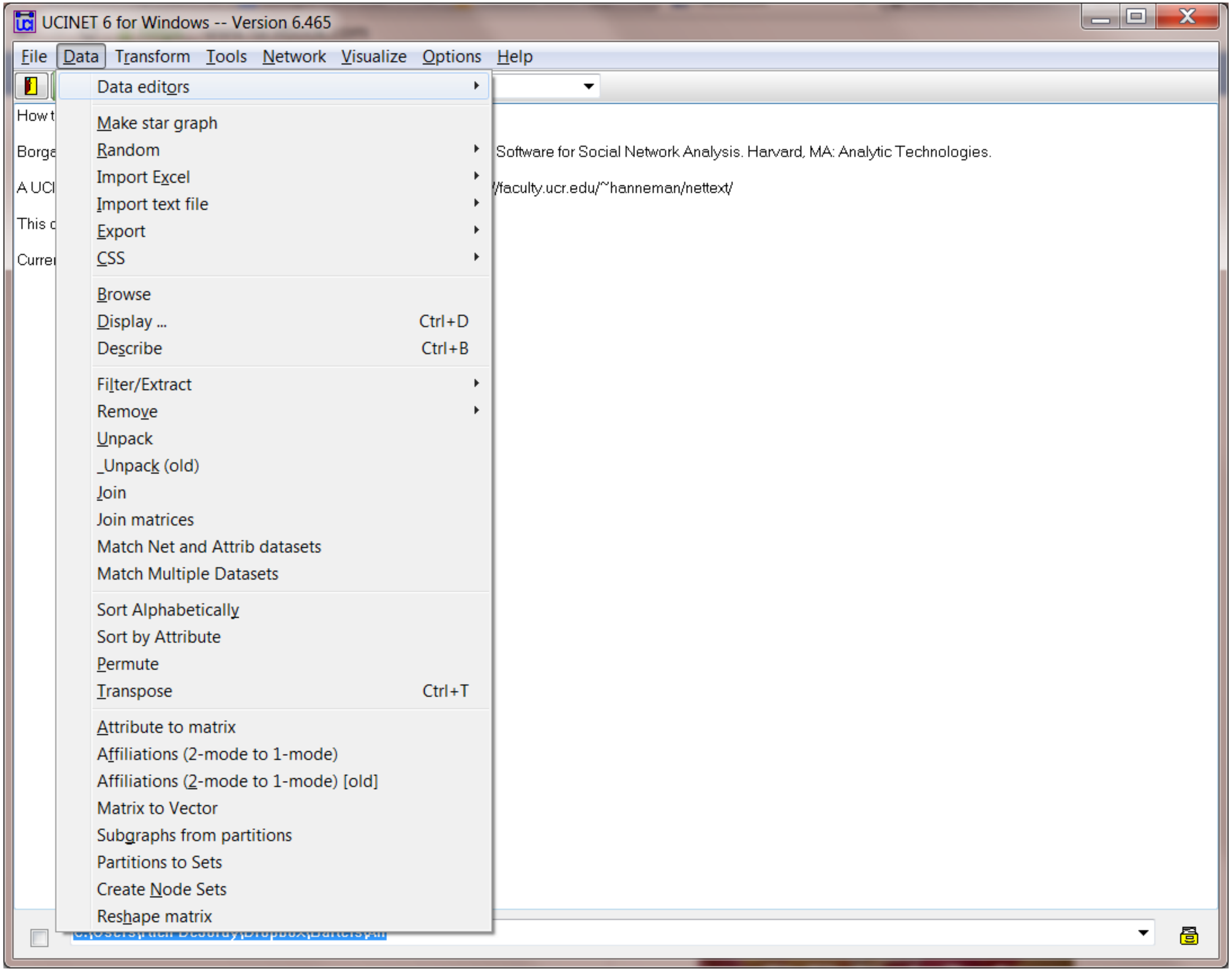
If you forget the format

- Just Export one of the Sample files
  - For DL files
    - From UCINET go to
      Data | Export | DL
  - For VNA files
    - From NetDraw, load the data and go to
      File | Save Data as | VNA | Complete

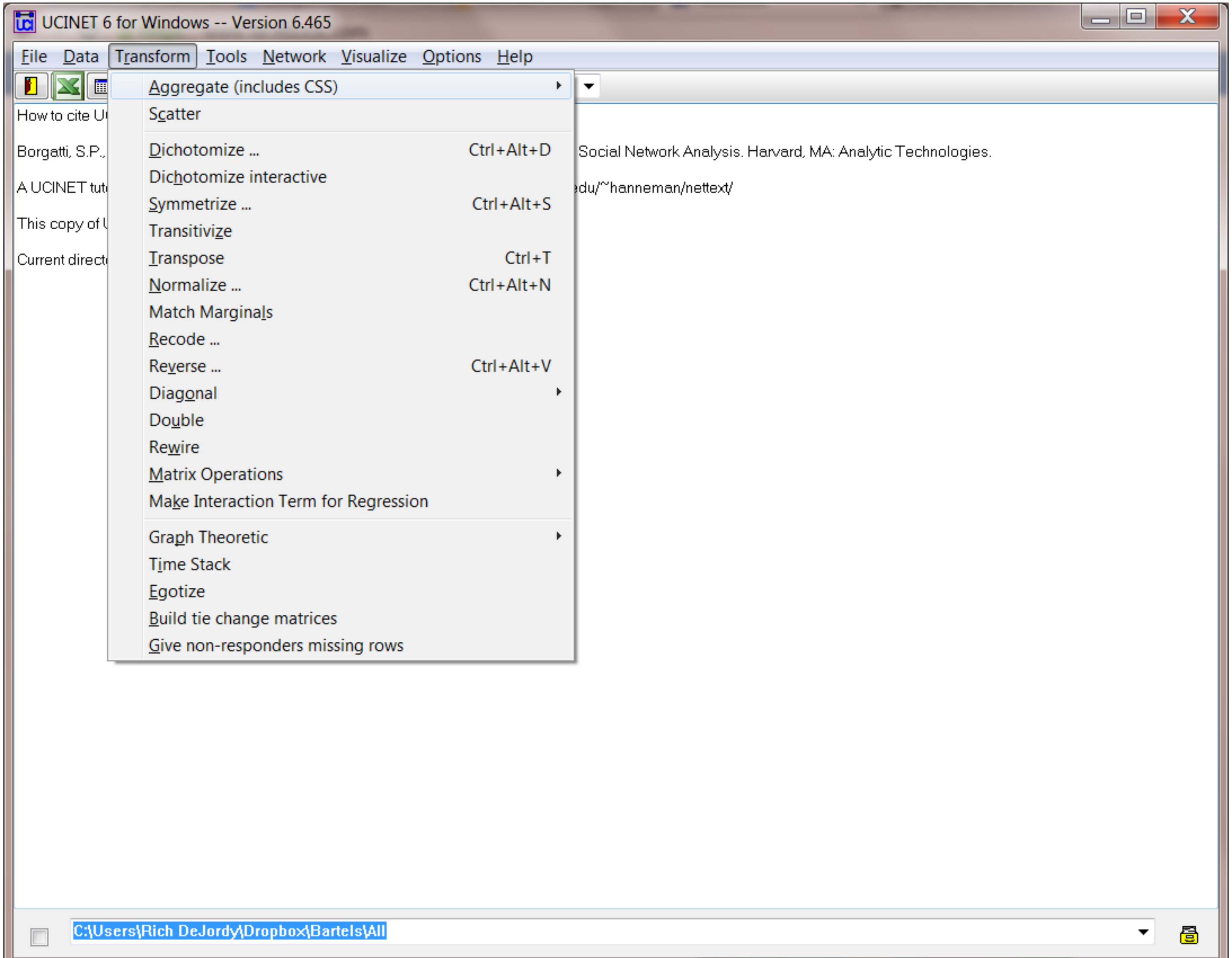
UCINET File Menu



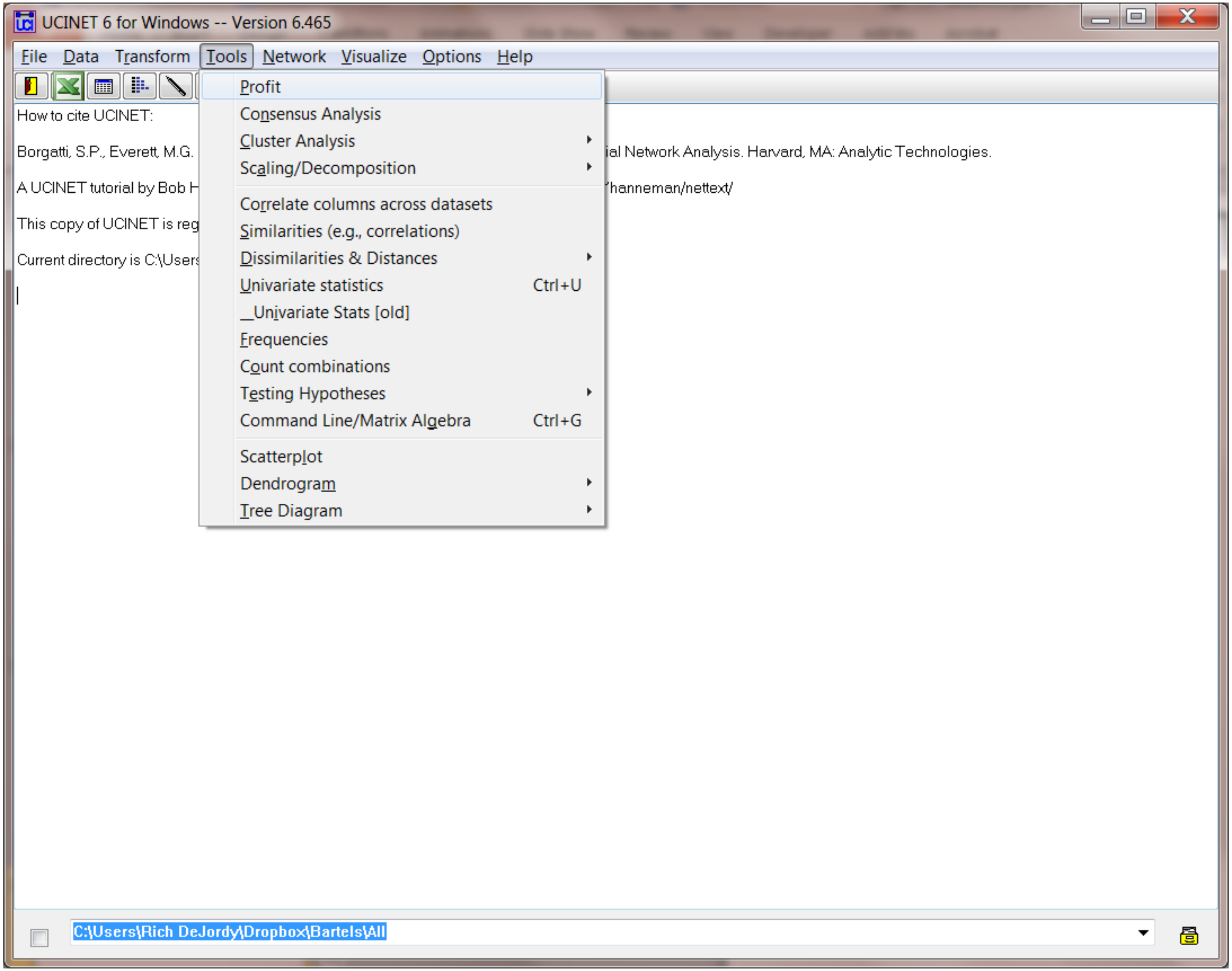
UCINET Data Menu



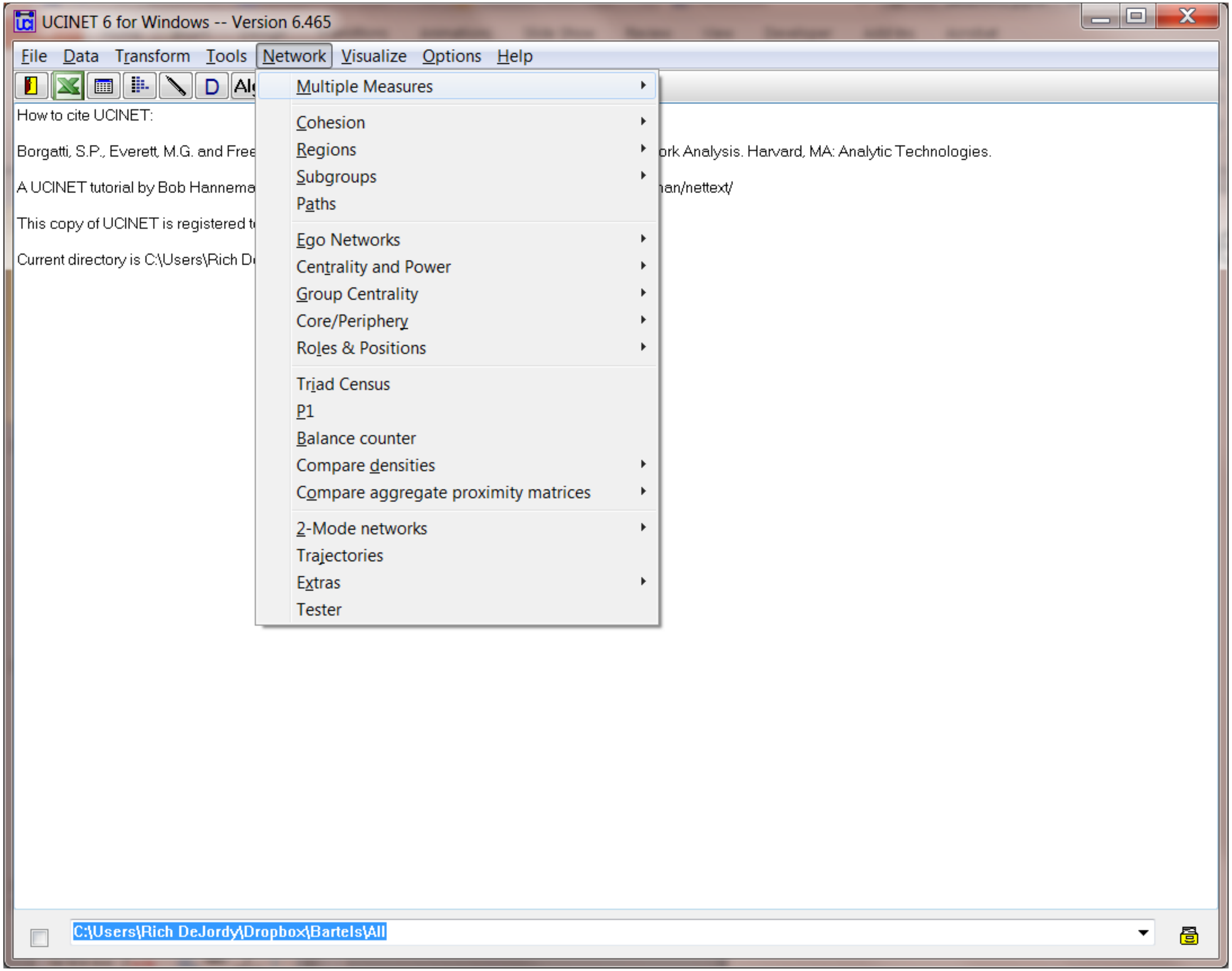
UCINET Transform Menu



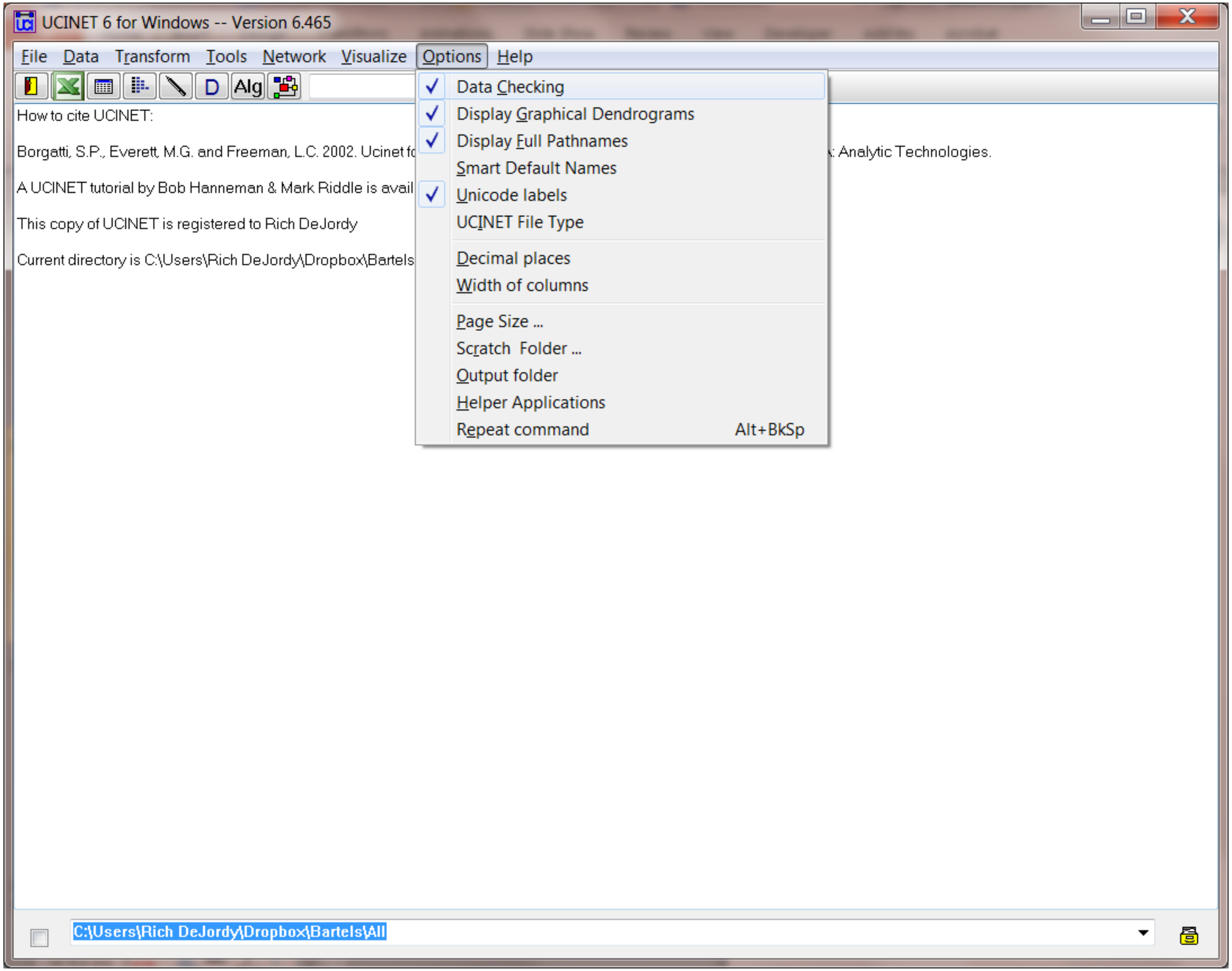
UCINET Tools Menu



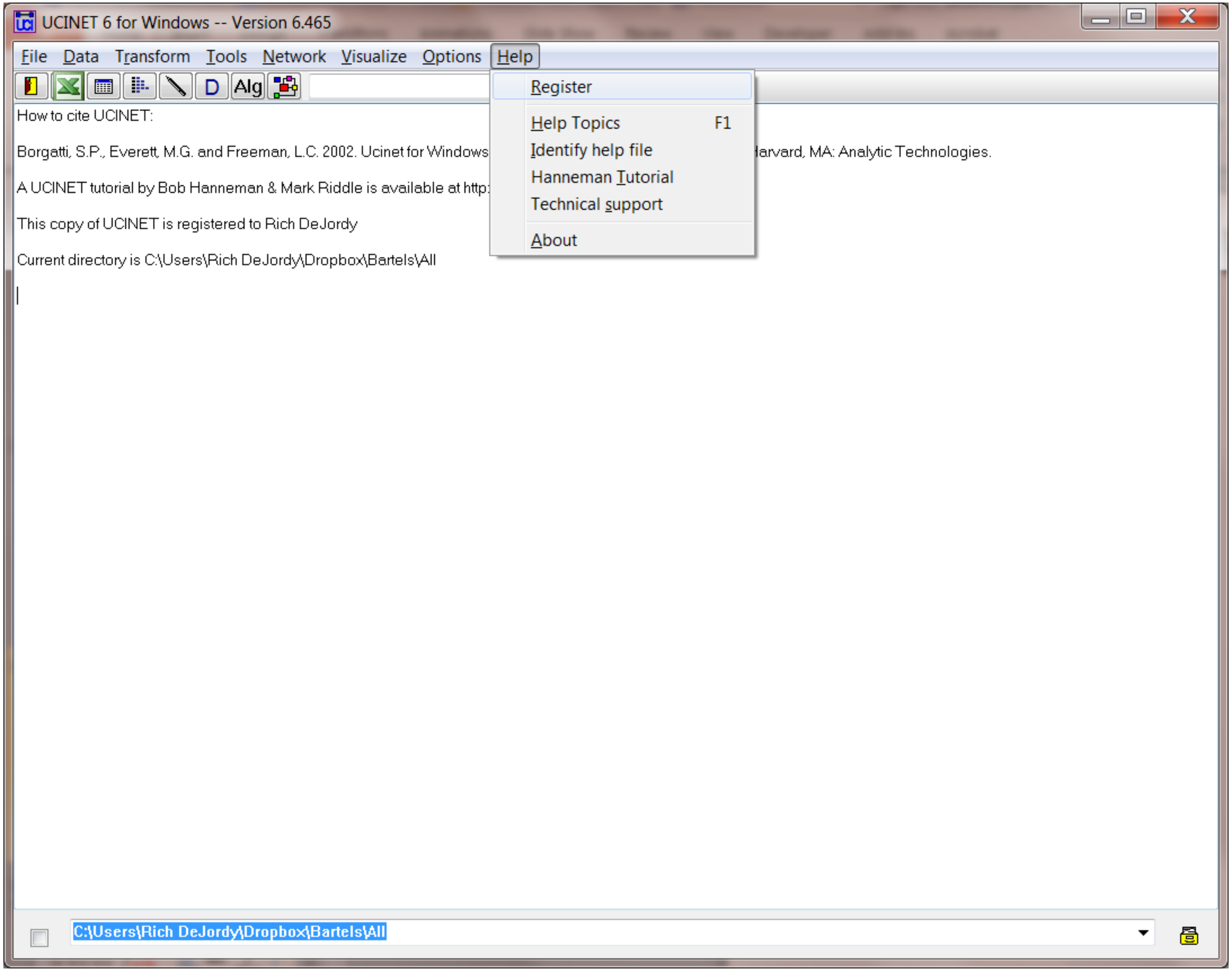
UCINET Network Menu



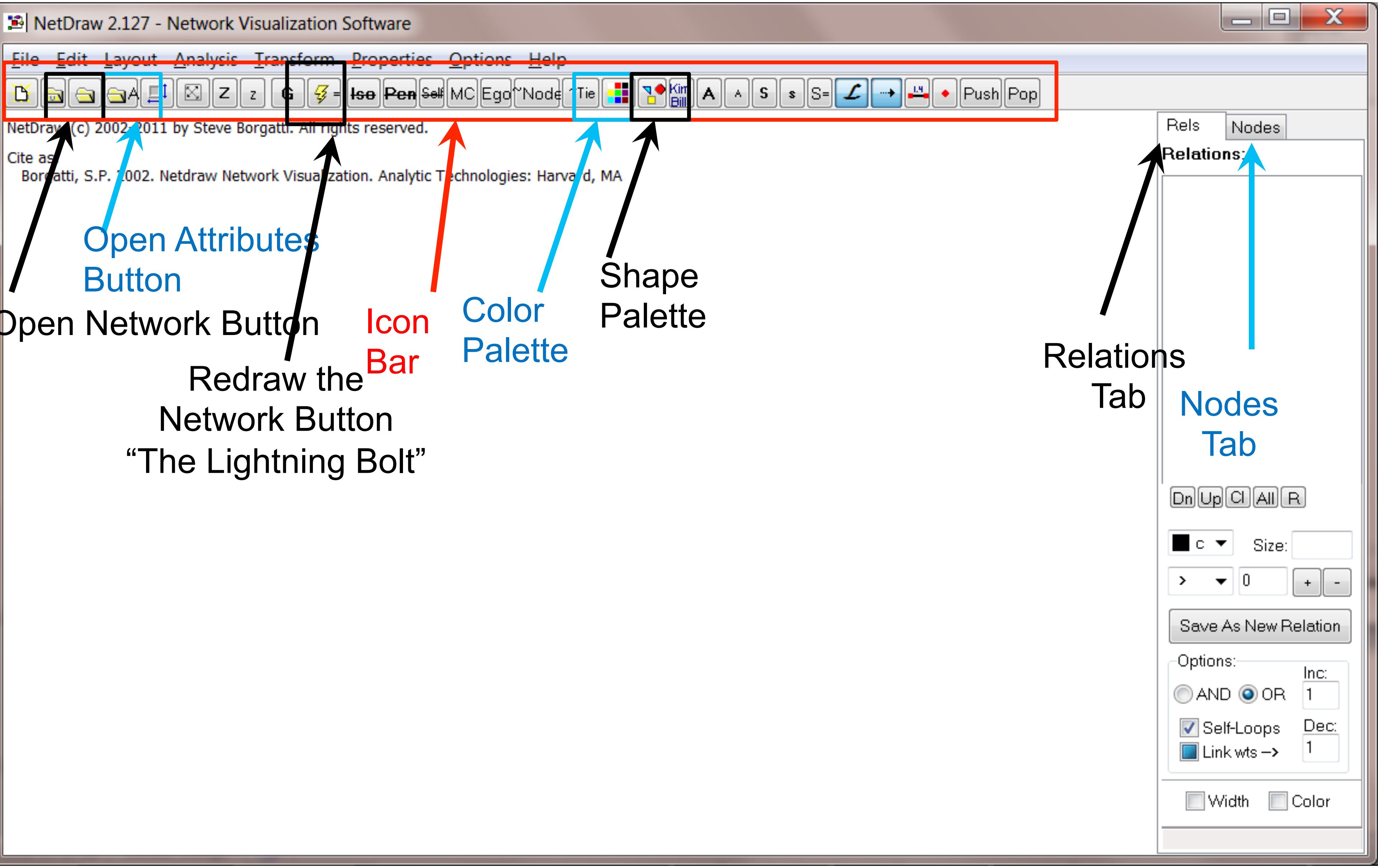
UCINET Options Menu



UCINET Help Menu



NetDraw


NetDraw 2.127 - Network Visualization Software
File Edit Layout Analysis Transform Properties Options Help
NetDraw (c) 2002-2011 by Steve Borgatti. All rights reserved.
Cite as
Borgatti, S.P. 2002. Netdraw Network Visualization. Analytic Technologies: Harvard, MA
Open Attributes Button
Open Network Button
Redraw the Network Button "The Lightning Bolt"
Icon Bar
Color Palette
Shape Palette
Relations Tab
Nodes Tab
Rels
Nodes
Relations:
Dn Up Cl All R
Size:
Save As New Relation
Options:
AND OR
Inc:
Self-Loops
Dec:
Link wts ->
Width
Color

Big 4 or 5 centrality measures

- Degree
- Closeness
- Betweenness
- Eigenvector / beta centrality

networks are complex

Can we understand them better without a “ridiculogram”?

describing networks

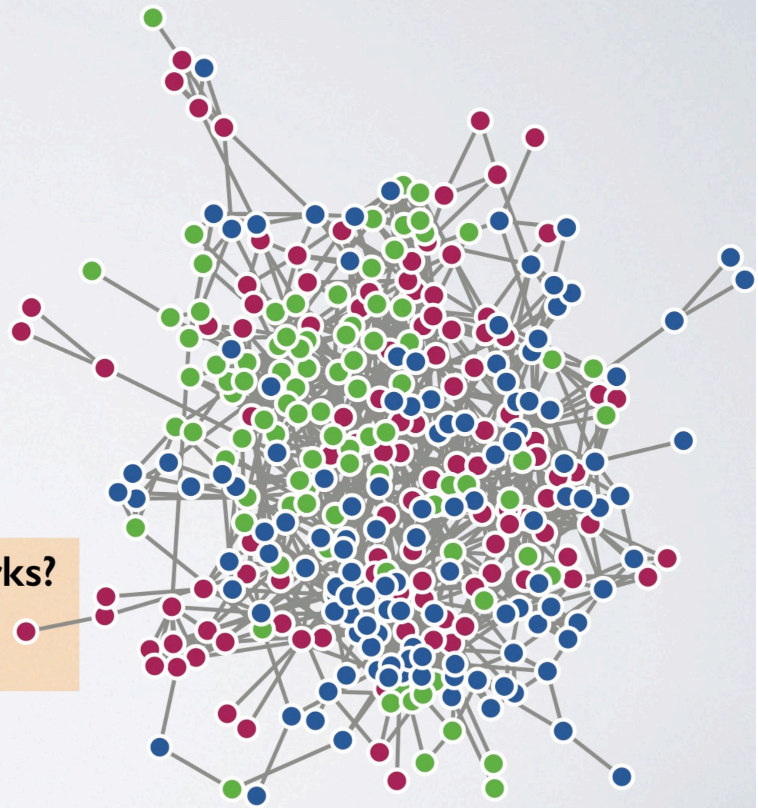
what networks look like

questions:

- **how are the edges organized?**
- **how do vertices differ?**
- **does network location matter?**
- **are there underlying patterns?**

what we want to know

- **what processes shape these networks?**
- **how can we tell?**

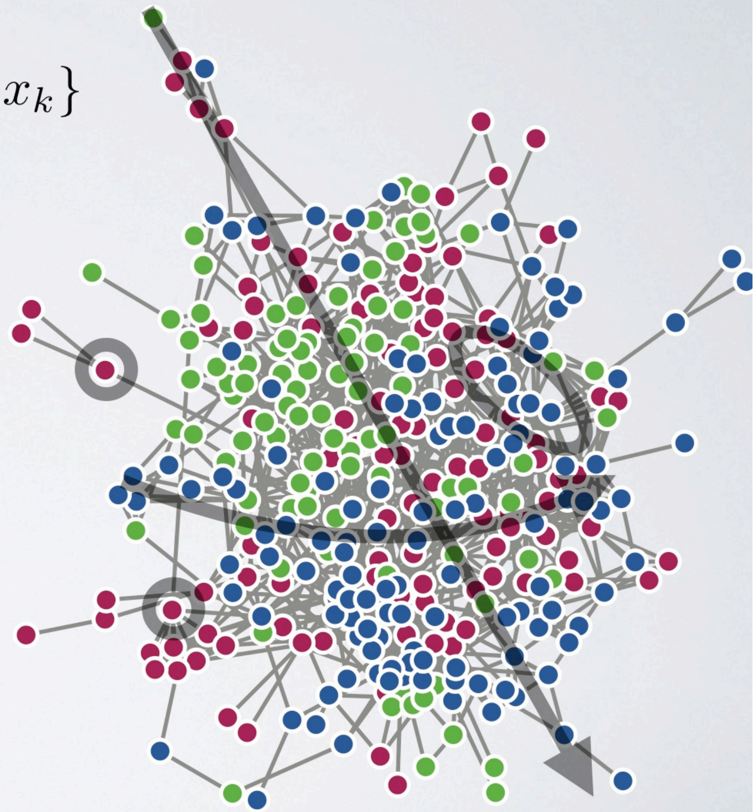


describing networks

a first step : **describe its features**

$$f : G \rightarrow \{x_1, \ldots, x_k\}$$

- **degree distributions**
- **short-loop density (triangles, etc.)**
- **shortest paths (diameter, etc.)**
- **vertex positions**
- **correlations between these**



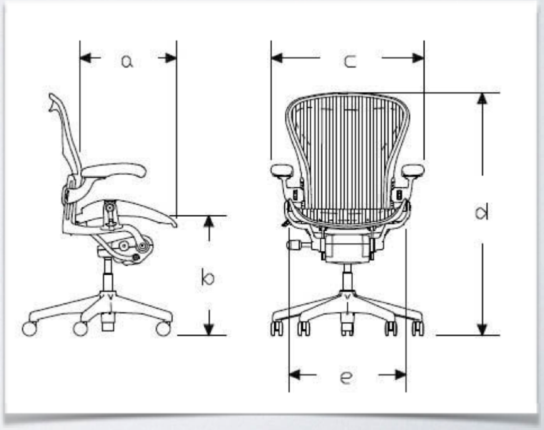
describing networks

a first step : **describe its features**

$$f : \text{object} \to \{x_1, \ldots, x_k\}$$

- **physical dimensions**
- **material density, composition**
- **radius of gyration**
- **correlations between these**

helpful for exploration, but not what we want…



describing networks

what we want : **understand its structure**

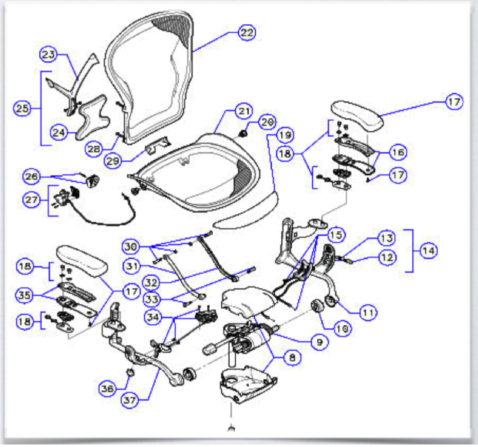
$$f : \text{object} \rightarrow \{\theta_1, \ldots, \theta_k\}$$

- **what are the fundamental parts?**
- **how are these parts organized?**
- **where are the degrees of freedom $\vec{\theta}$?**
- **how can we define an abstract class?**
- **structure — dynamics — function?**

what does ***local-level structure*** look like?

what does ***large-scale structure*** look like?

how does ***structure constrain*** function?



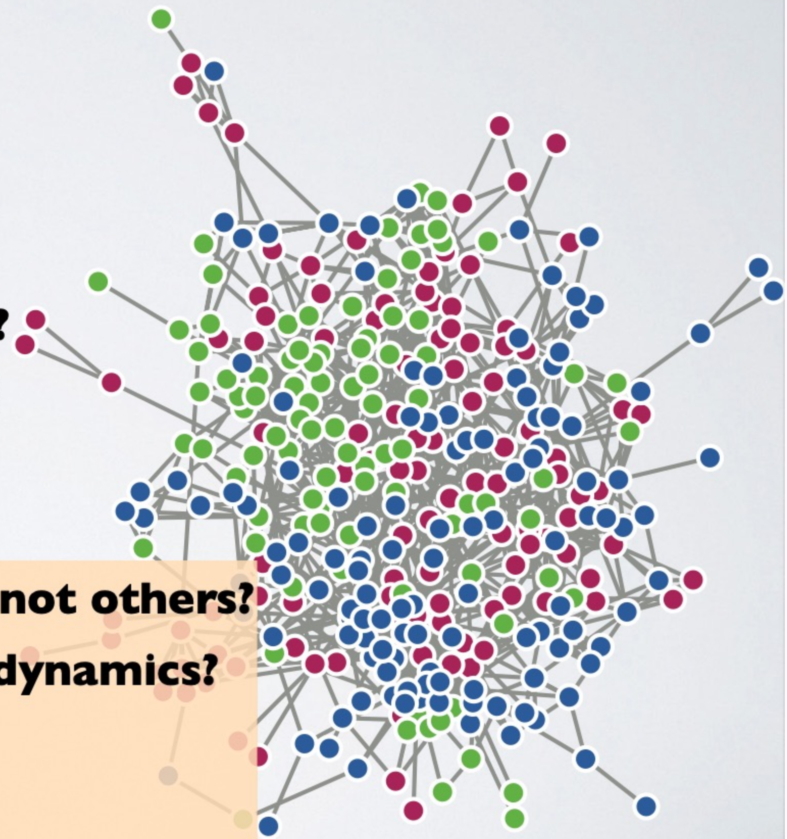
describing networks

what real networks look like…

questions:

descriptive questions

- **how are the edges organized?**
- **how do vertices differ?**
- **does network location matter?**
- **are there underlying patterns?**



what we want to know

process questions

- **why do some edges exist, and not others?**
- **how does structure constrain dynamics?**
- **what does structure predict?**
- **how can we tell?**

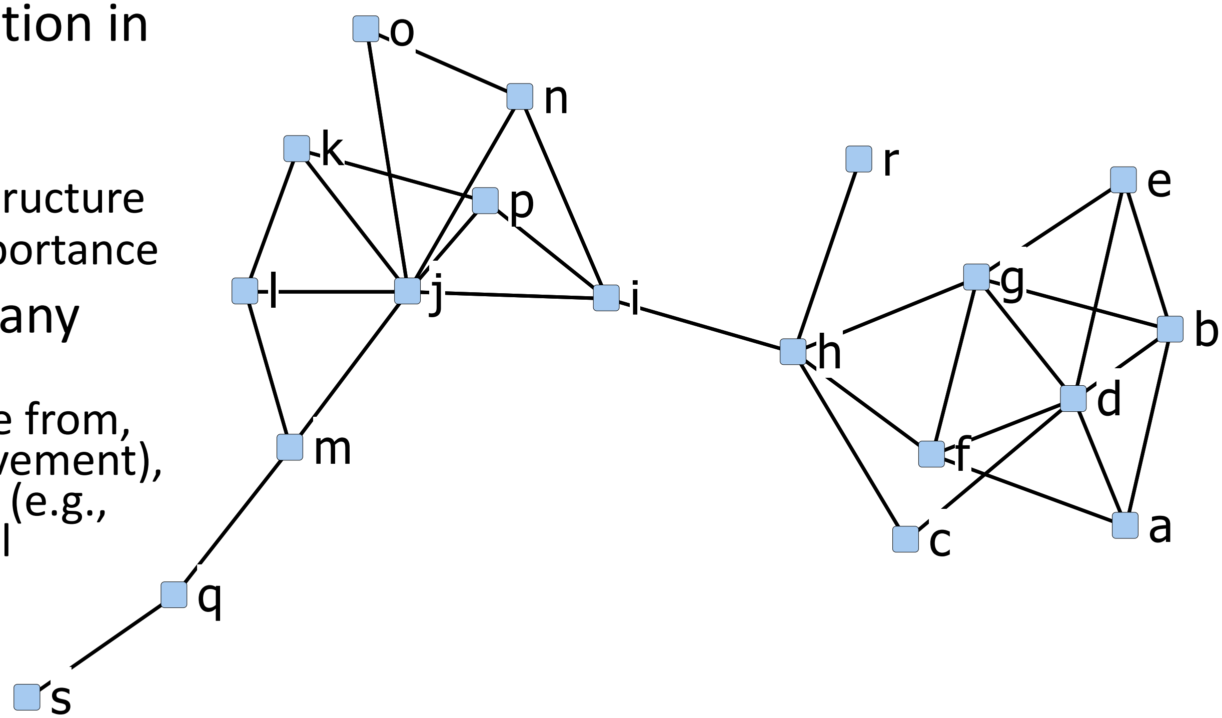
describing networks

aka, summarizing a network's structure

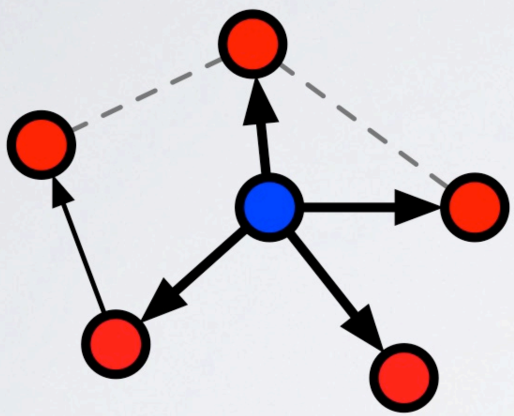
$$f : G \rightarrow \underbrace{\{x_1, \ldots, x_k\}}_{\textbf{summary statistics}}$$

What is centrality?

- An aspect of a node's position in a network
- Structural prominence
  - Contribution to network structure
  - Structural reflection of importance
- Direction of a causality in any context is often unclear
  - Does central position come from, node attribute (e.g., achievement), or does the node attribute (e.g., disease) come from central position?
- Measures or constructs?



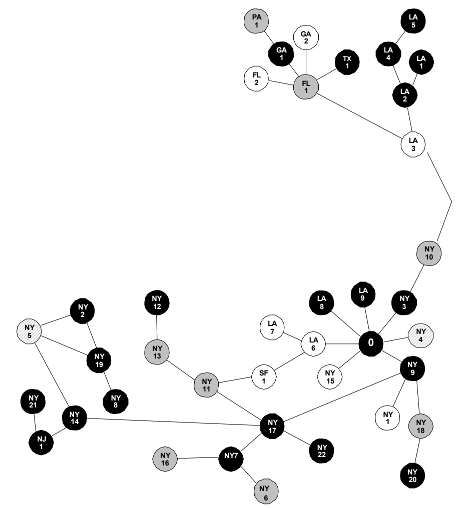
describing networks



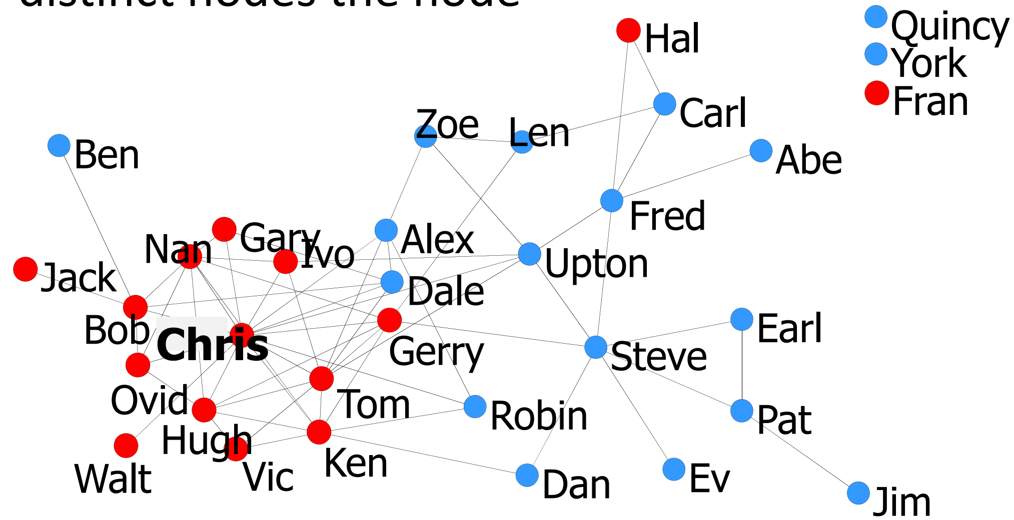
position = centrality:
measure of positional “importance”

geometric	harmonic centrality	
	closeness centrality	
	betweenness centrality	
connectivity	degree centrality	
	eigenvector centrality	Algebraic Centrality
	PageRank	
	Katz centrality	
	many many more…	

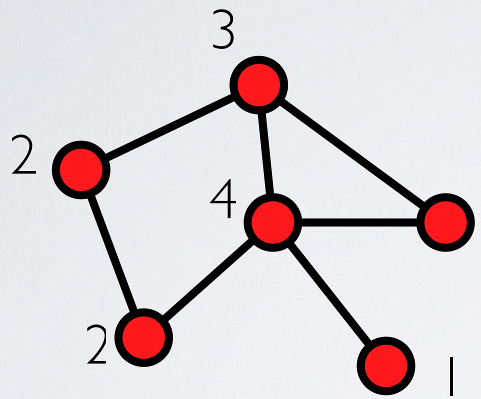
Degree centrality



- Barely a centrality measure, as you don't need to know the structure of the network to calculate it
- Number of ties a node has
  - In most cases, this is also number of distinct nodes the node is adjacent to
- Interpreted as exposure and capacity to influence
  - Depending on the tie
    - E.g., negative ties work differently



describing networks



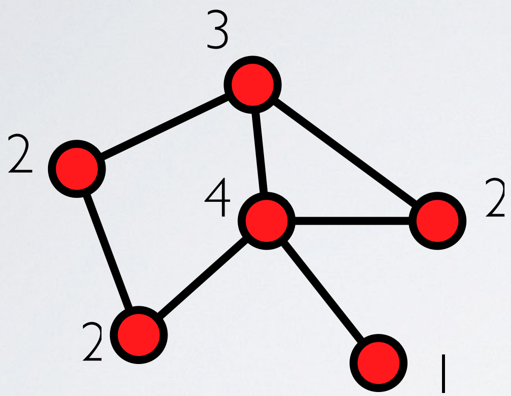
degree:
number of connections k

$$k_i = \sum_j A_{ij}$$

number of edges $$m = \frac{1}{2}\sum_{i=1}^{n} k_i = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij} = \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n} A_{ji}$$

mean degree $$\langle k \rangle = \frac{1}{n}\sum_{i=1}^{n} k_i = \frac{2m}{n}$$

describing networks



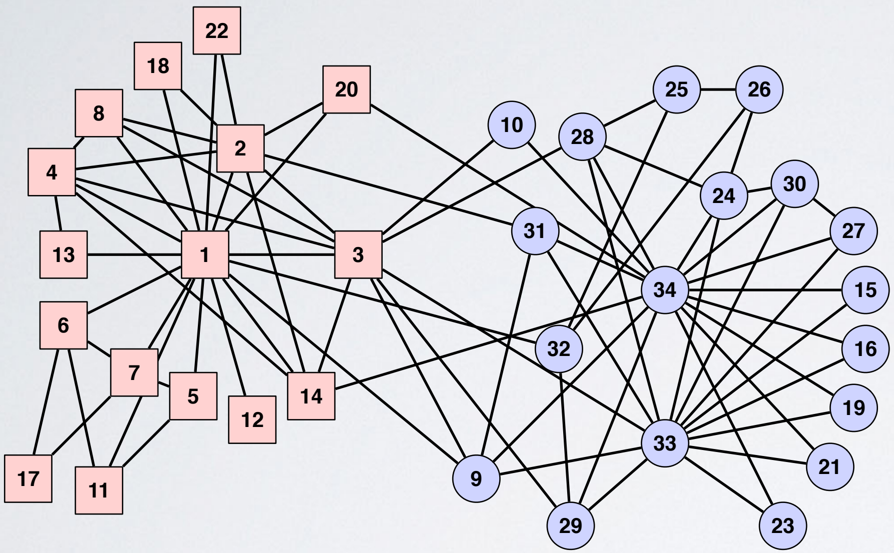
degree:
number of connections k

$$k_i = \sum_j A_{ij}$$

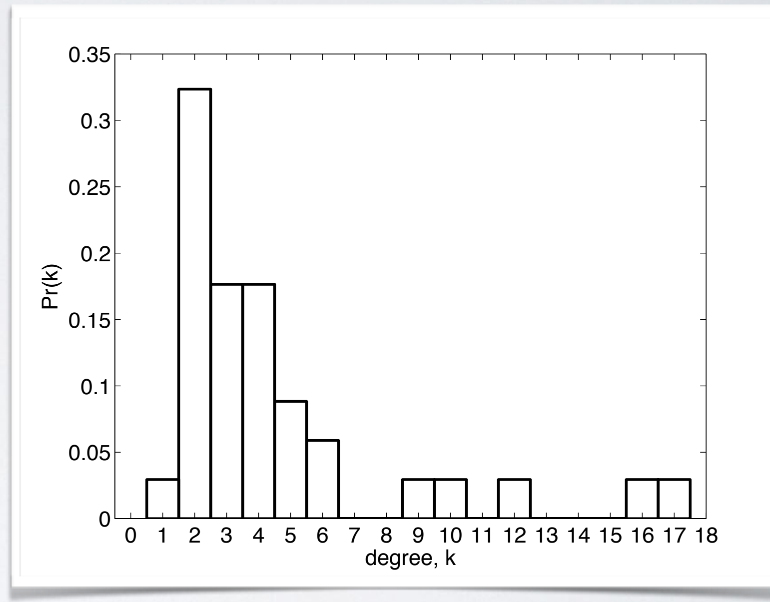
degree sequence $\{1, 2, 2, 2, 3, 4\}$

degree distribution $\Pr(k) = \left[\left(1, \frac{1}{6}\right), \left(2, \frac{3}{6}\right), \left(3, \frac{1}{6}\right), \left(4, \frac{1}{6}\right)\right]$

describing networks



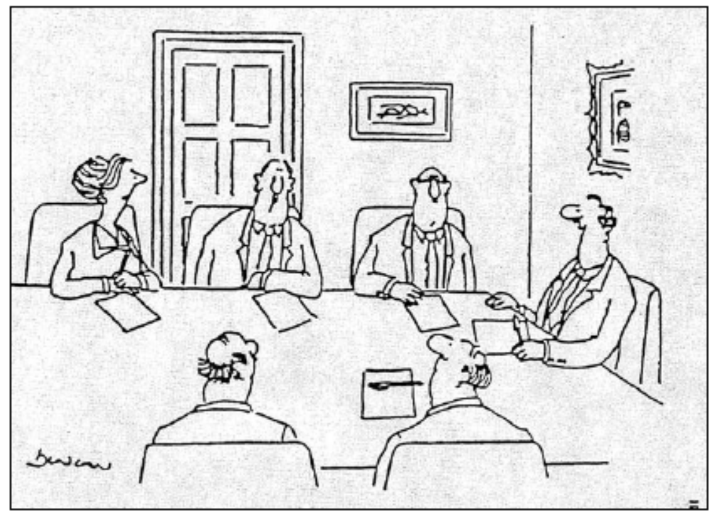
Zachary karate club*



Centrality in social context

- Social capital
  - The more ties I have, the more potential help I can get for some problem
  - Also the greater the likelihood that some node close to me has a needed skill/resource
- Power – influencing others
  - Imagine mutual trust ties
  - The more ties, the more people you can influence directly
- Adoption – being influenced by others
  - If you have many trust ties, lots of people have influence on you.
  - What if your alters disagree with each other?
    - Role strain/cognitive dissonance

- Reflection of status/visibility in another context

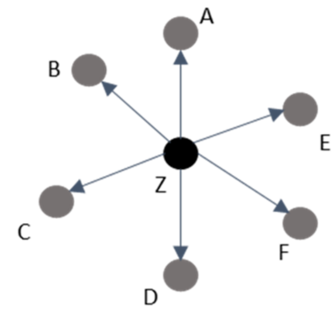
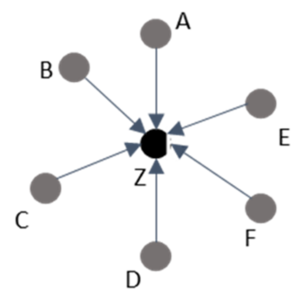


"That's an excellent suggestion, Miss Triggs. Perhaps one of the men here would like to suggest it." (Punch, 8 January, 1988)

Effects of degree depend on the kinds of ties being considered

Dark side of centrality?

Independent Variable: Task Interdependence Networks

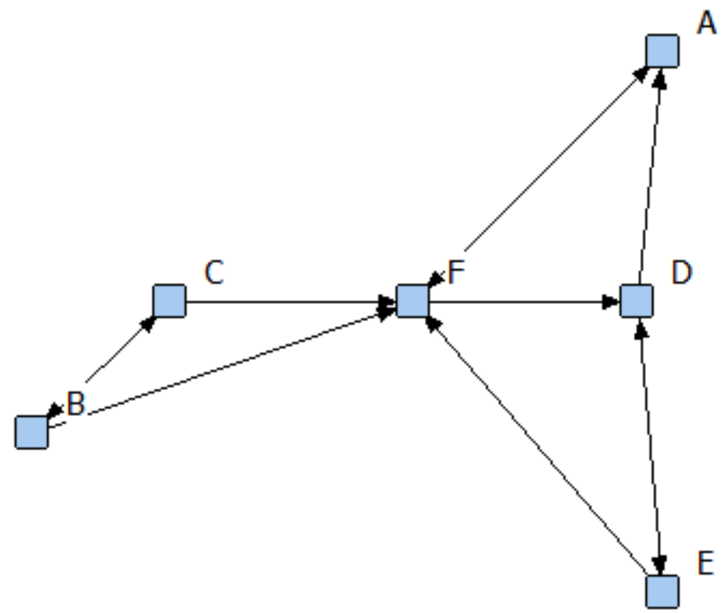
		DV
H1a: The higher the number of contacts that an actor depends on to develop his/her own work, the more likely he/she is to experience high cynicism and low professional efficacy		- Cynicism - Professional Efficacy
H1b: The higher the number of contacts that depend on an actor to develop their own work, the more likely he/she is to experience emotional exhaustion.		- Emotional Exhaustion

Shamelessly lifted from the dissertation proposal "Influence of Social Relations at Work on the Process of Development of Burnout Syndrome: A Longitudinal Study" by Camila Umaña

Directed degree – cont.

- Outdegree and indegree correspond to the row and column sums of the adjacency matrix
  - Outdegree = row sums
  - Indegree = column sums

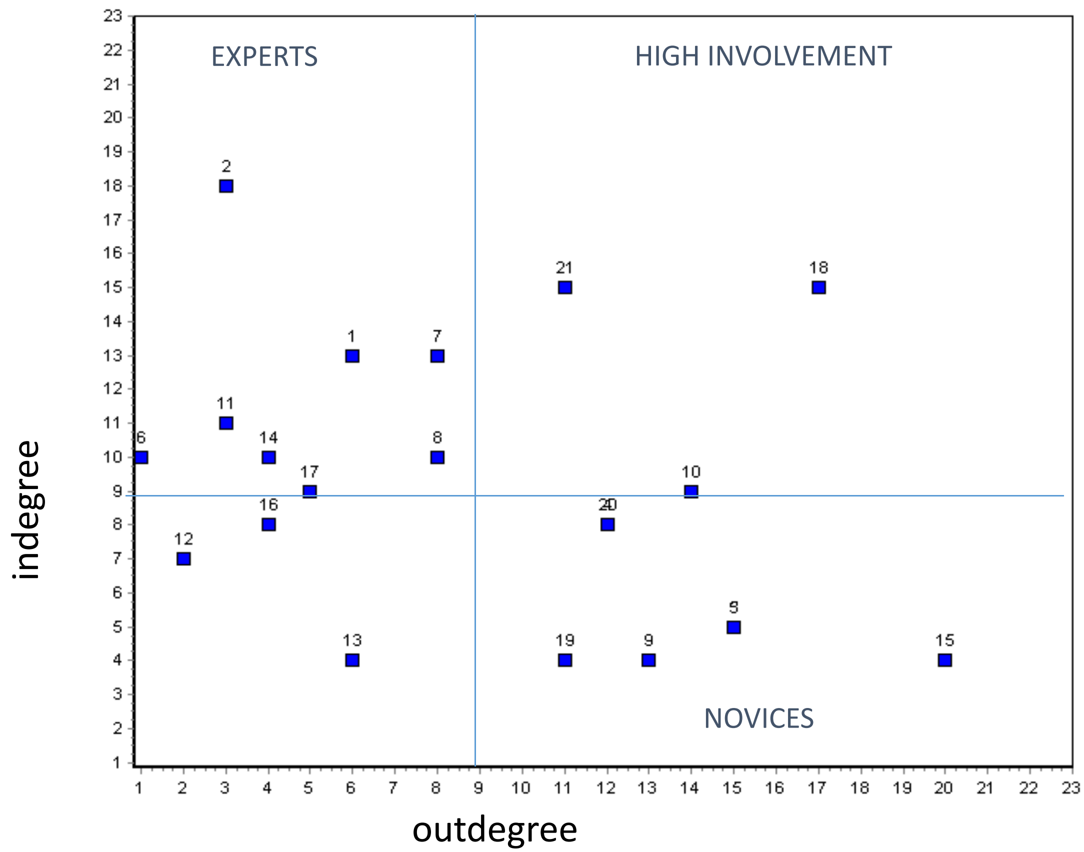
	A	B	C	D	E	F	Total
A	0	0	0	0	0	1	1
B	0	0	1	0	0	1	2
C	0	1	0	1	0	1	3
D	1	0	0	0	1	0	2
E	0	0	0	1	0	1	2
F	1	0	0	1	0	0	2
Total	2	1	1	3	1	4	12

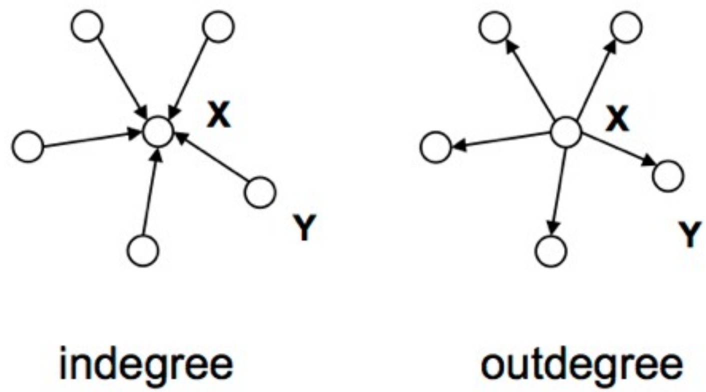


Plot indegree vs outdegree

->unpack krack-high-tec
->deg = degree(advice)
->excel deg

- Suppose the type of tie is "seeks advice from"
  - Outdegree = how many people you seek help from
  - Indegree = how many people seek advice from you
- Note that flow of information runs backwards: if A seeks advice from B, then B sends info to A





Best measure if importance means:

→ how popular you are

→ how many people you know

It is a local measure!

Degree Centrality with Valued Data

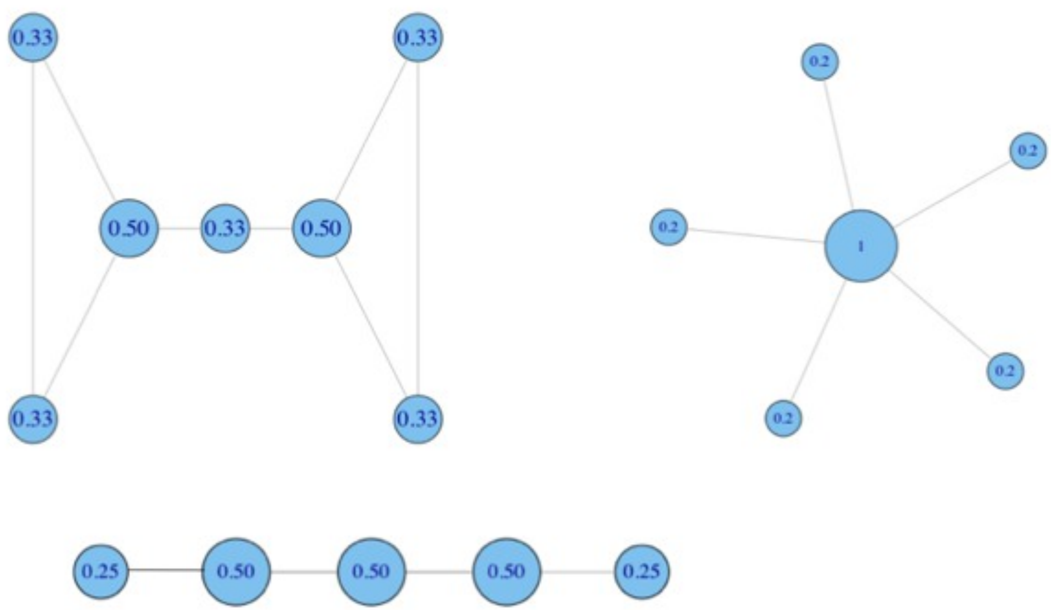
OUTDEGREE

	MT6	MT71	MT72	MT83	MT93	MT210	MT215	MT272	
MT6	0	100	500	1600	1100	300	2450	1500	7550
MT71	0	0	0	0	0	0	0	0	0
MT72	0	0	0	0	0	0	0	0	0
MT83	0	0	0	0	0	0	0	0	0
MT93	0	0	0	0	0	0	0	0	0
MT210	0	0	0	0	0	0	0	0	0
MT215	0	0	0	0	0	0	0	0	0
MT272	0	0	0	0	0	0	0	0	0
INDEGREE	0	100	500	1600	1100	300	2450	1500	0

NOTE: some software may binarize networks before calculating degree with valued data.

formula for degree (normalized)

$$C^D(i) = \frac{k_i}{N-1}$$



describing networks

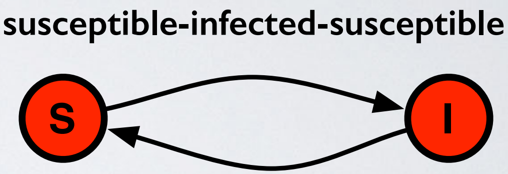
spreading processes on networks

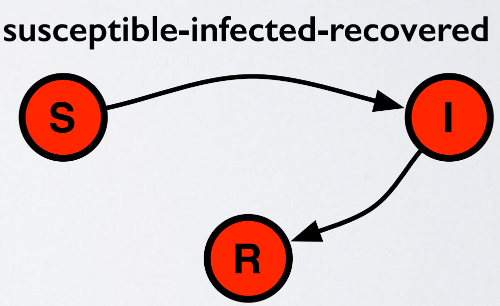
biological (diseases)

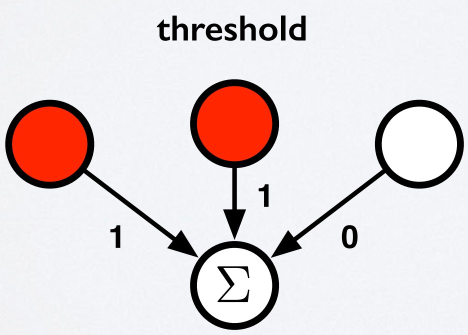
- SIS and SIR models

social (information)

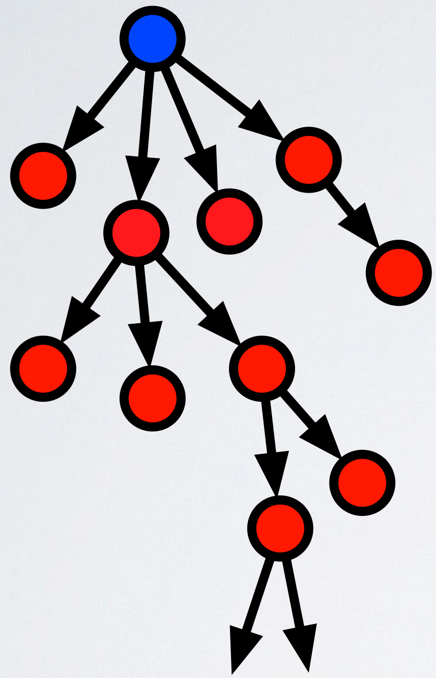
- SIS, SIR models
- threshold models







describing networks



$R_0 = 0.923\ldots$

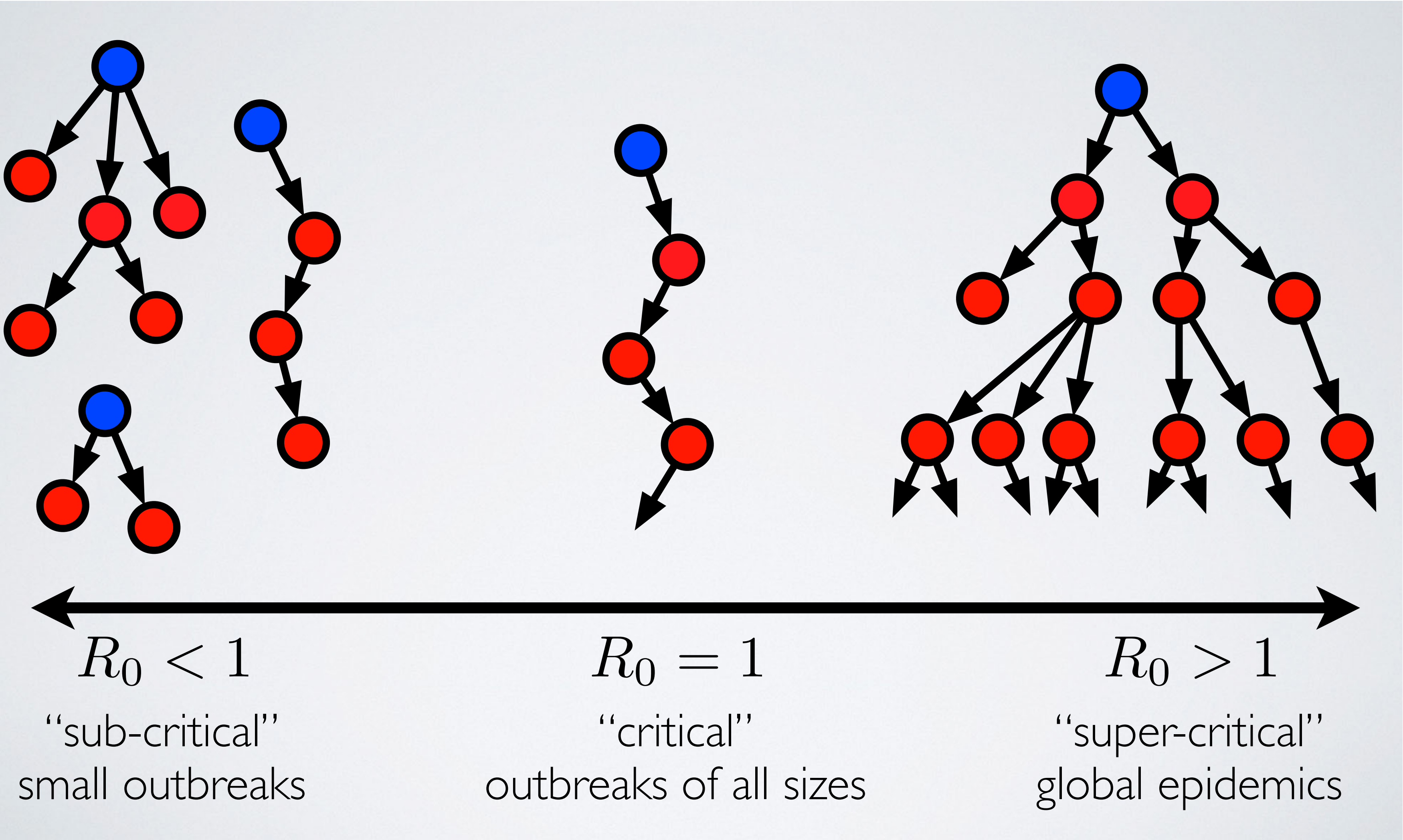
R_0 is the basic reproduction number: the number of infected people an infected person can *reproduce*.

cascade
epidemic
branching process
spreading process

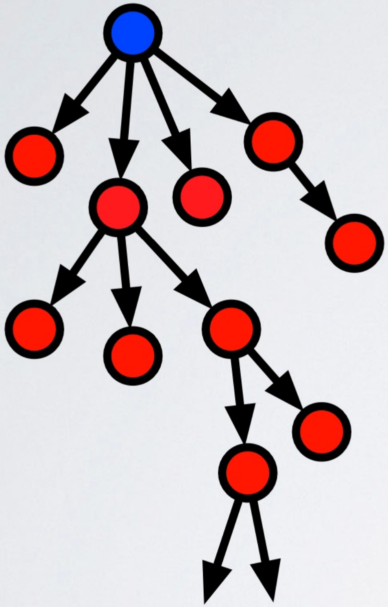
$$R_0 = \text{net reproductive rate} = \text{average degree } \langle k \rangle$$

caveat:
ignores network structure, dynamics, etc.

describing networks



describing networks

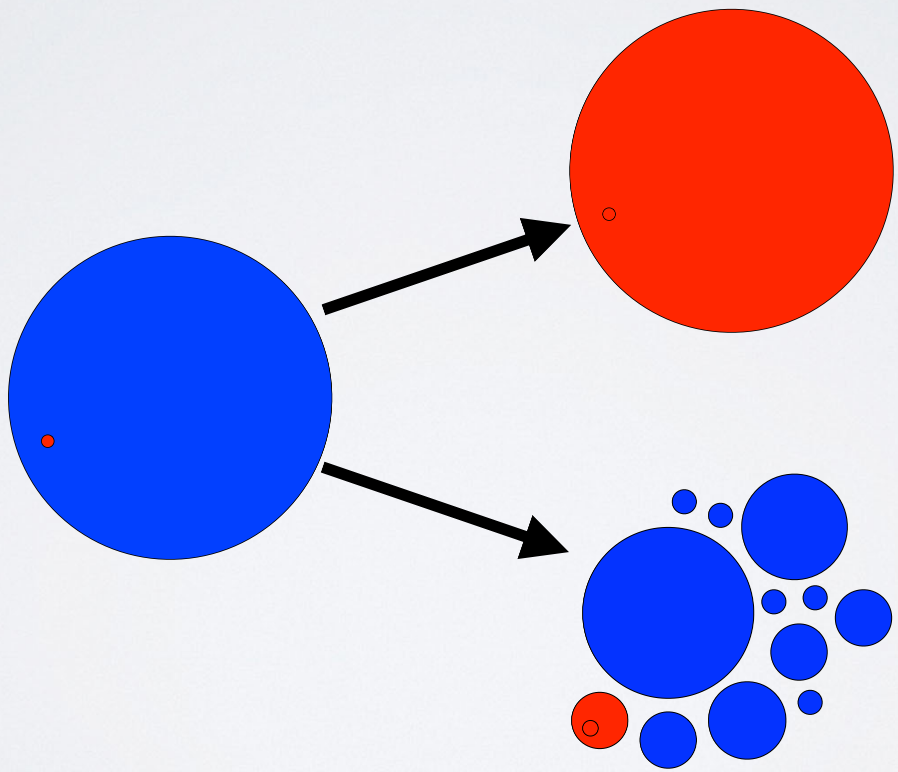


disease	R_0	transmission	vax.
measles	12 – 18	airborne	90 – 95%
chickenpox	7 – 12		85 – 90%
polio	5 – 7	fecal-oral route	82 – 87%
small pox	1.5 – 20+	airborne droplet	70 – 80%
H1N1 flu	1 – 3	airborne droplet	≈ 67%
ebola	1.5 – 2.5	bodily fluids	
zika	2		
covid-19 (wildtype)	≈ 2.4	aerosols	≈ 60%
covid-19 (alpha)	4 – 5	aerosols	75 – 80%
covid-19 (delta)	5 – 8	aerosols	80 – 88%
covid-19 (omicron)	10 – 14	aerosols	90 – 93%

all super-critical

how could we halt the spread?

- break network into disconnected pieces

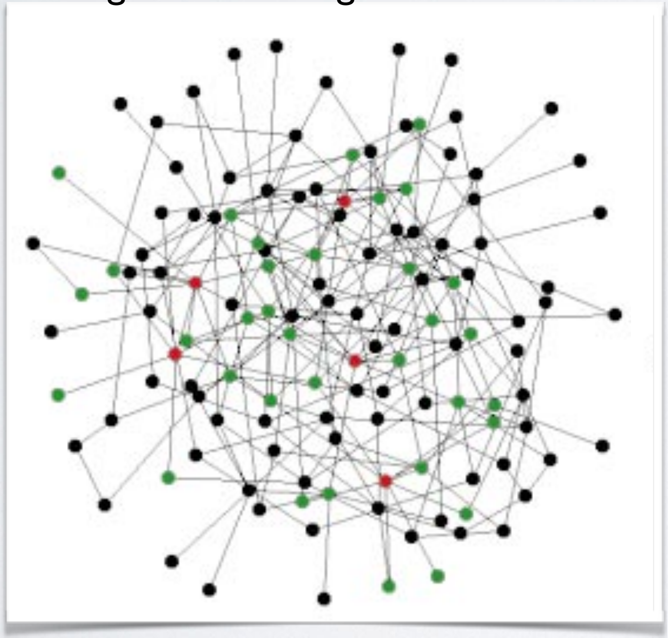


describing networks

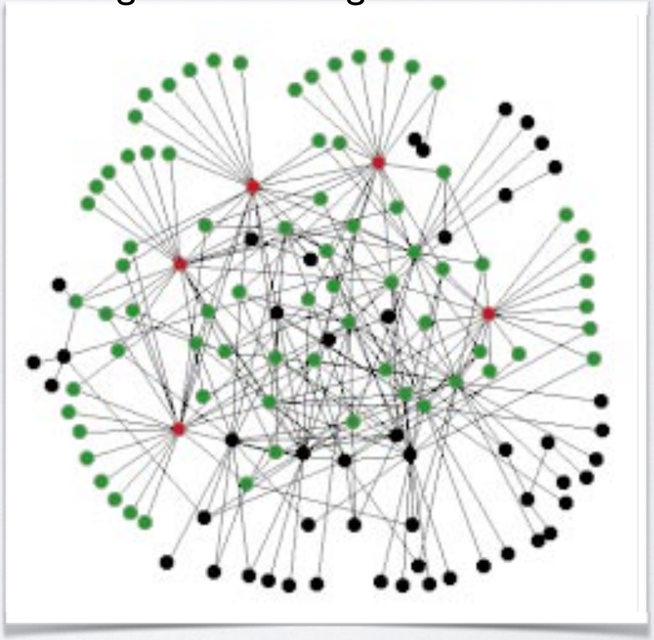
what promotes spreading?

- high-degree vertices*
- centrally-located vertices

homogeneous in degree

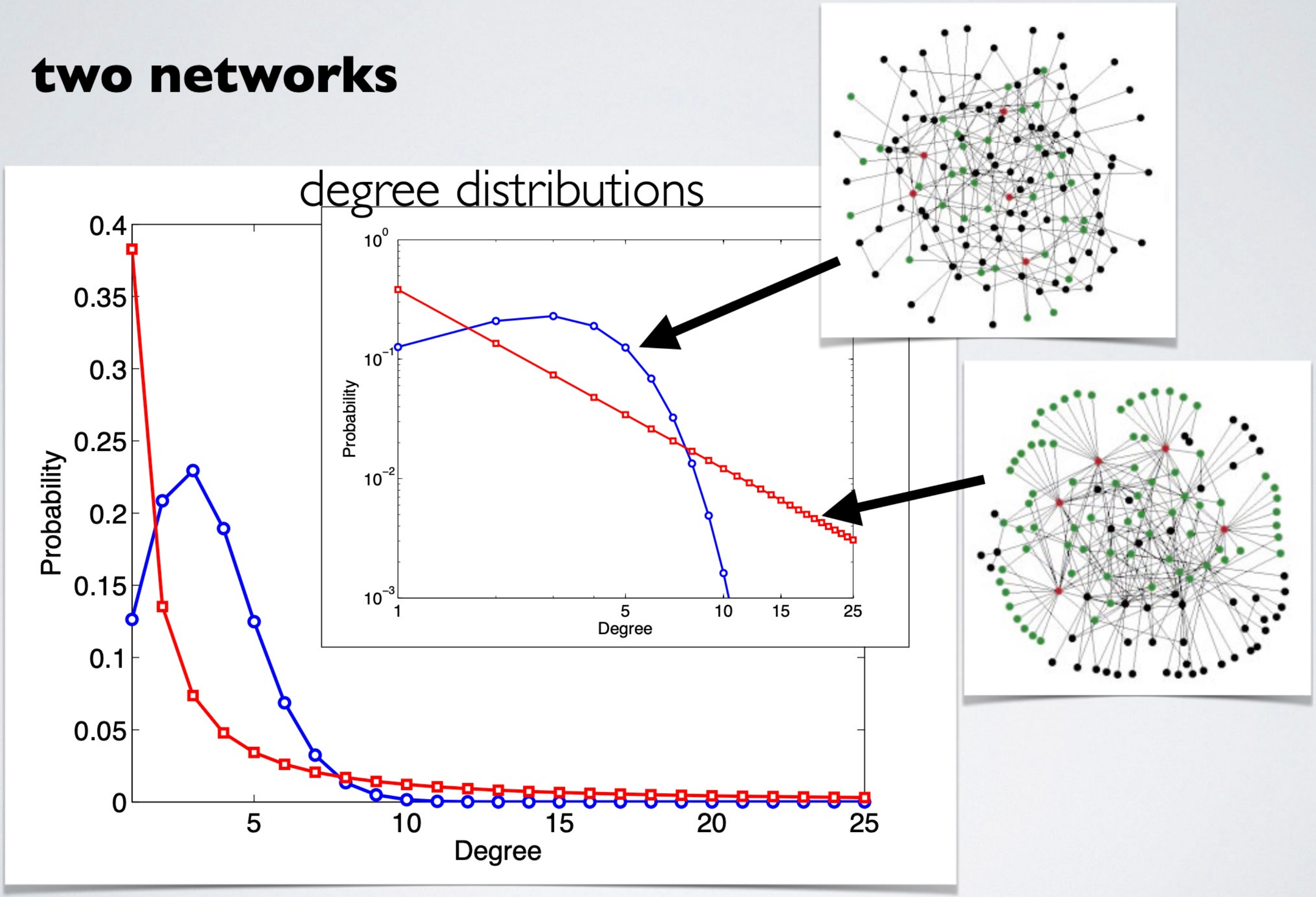


heterogeneous in degree

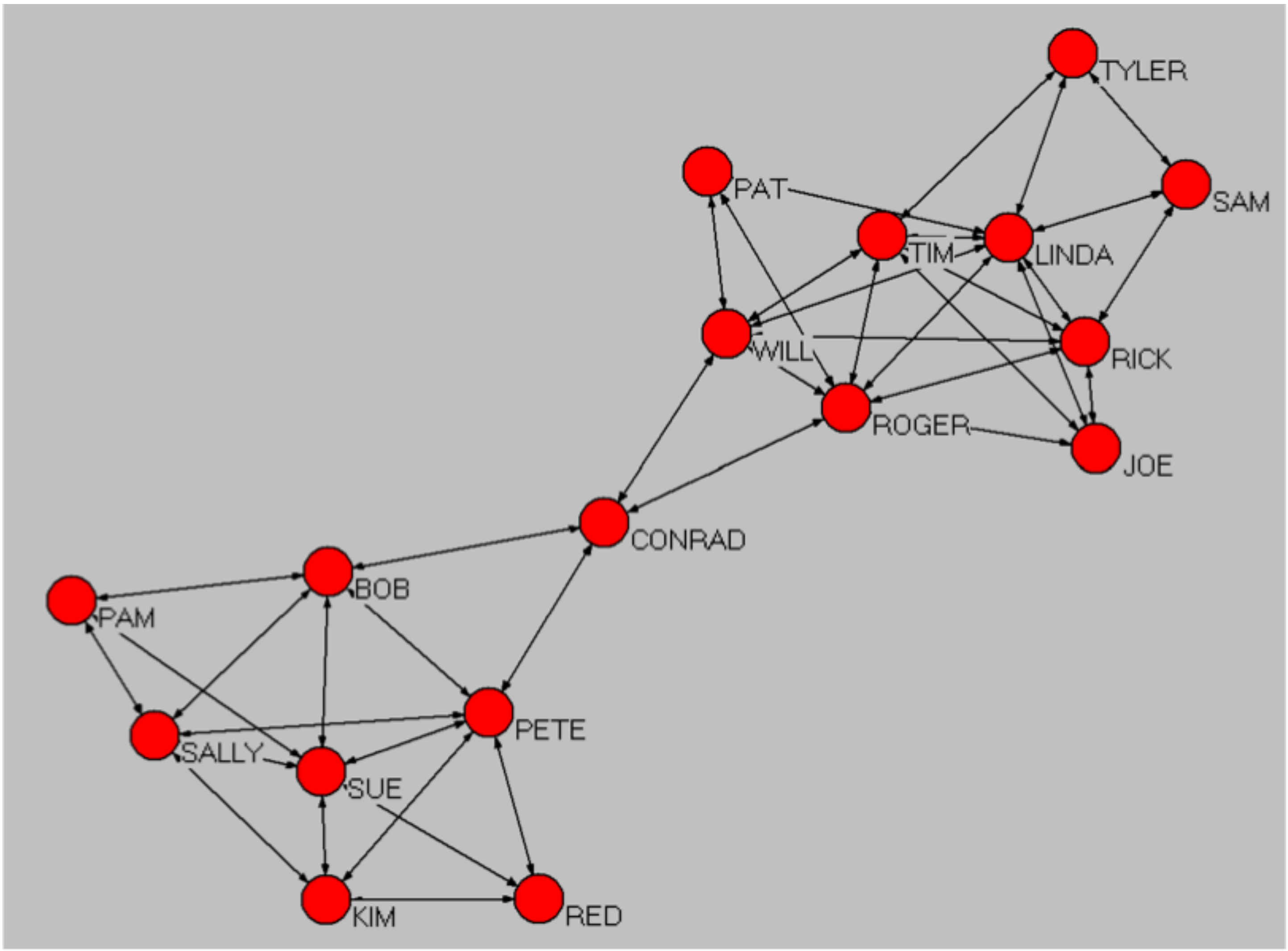


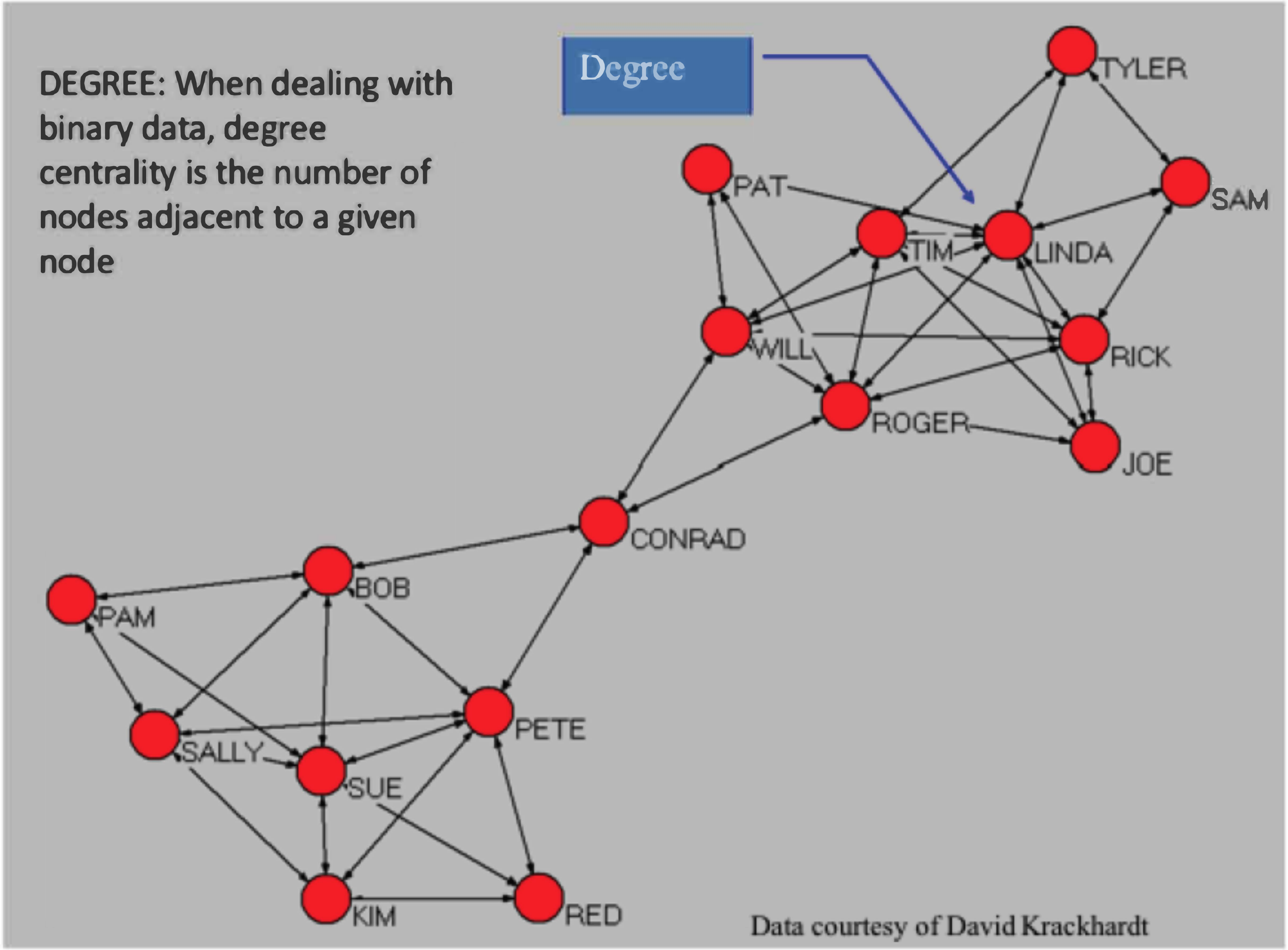
describing networks

two networks



Who's Important in this network?

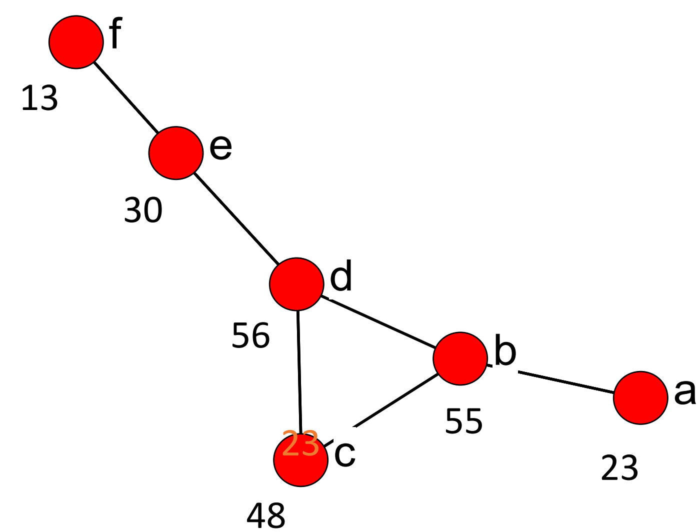



DEGREE: When dealing with binary data, degree centrality is the number of nodes adjacent to a given node
Degree
TYLER
PAT
SAM
TIM
LINDA
WILL
RICK
ROGER
JOE
CONRAD
BOB
PAM
PETE
SALLY
SUE
KIM
RED
Data courtesy of David Krackhardt

Turbo-charging degree

- Degree is a count of the number of nodes you are connected to
  - Treats all nodes equally
- What if you wanted to weight the nodes by how many nodes they were connected to?

$$td_i = \sum_j a_{ij} d_j$$



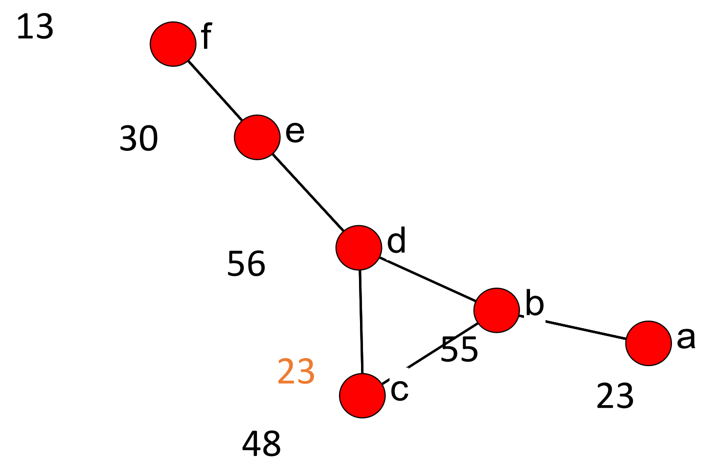
Turbo-charging degree

- Degree is a count of the number of nodes you are connected to
  - Treats all nodes equally
- What if you wanted to weight the nodes by how many nodes they were connected to?

$$td_i = \sum_j a_{ij} d_j$$

- But why stop there? Can keep iterating …

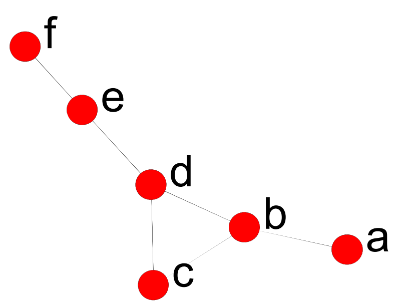
	a	b	c	d	e	f	deg	wtd deg
a	**0**	**1**	**0**	**0**	**0**	**0**	1	3
b	**1**	**0**	**1**	**1**	**0**	**0**	3	6
c	**0**	**1**	**0**	**1**	**0**	**0**	2	6
d	**0**	**1**	**1**	**0**	**1**	**0**	3	7
e	**0**	**0**	**0**	**1**	**0**	**1**	2	4
f	**0**	**0**	**0**	**0**	**1**	**0**	1	2



Iterated Degree

	a	b	c	d	e	f	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
a	0	1	0	0	0	0	1	3	6	16	35	86	195	465	1071	2524
b	1	0	1	1	0	0	3	6	16	35	86	195	465	1071	2524	5854
c	0	1	0	1	0	0	2	6	13	32	73	173	401	940	2190	5117
d	0	1	1	0	1	0	3	7	16	38	87	206	475	1119	2593	6086
e	0	0	0	1	0	1	2	4	9	20	47	107	253	582	1372	3175
f	0	0	0	0	1	0	1	2	4	9	20	47	107	253	582	1372
							12	28	64	150	348	814	1896	4430	10332	24128

ratio of differences stabilizes in latter iterations



	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
A	8.3	10.7	9.4	10.7	10.1	10.6	10.3	10.5	10.4	10.5
B	25.0	21.4	25.0	23.3	24.7	24.0	24.5	24.2	24.4	24.3
C	16.7	21.4	20.3	21.3	21.0	21.3	21.1	21.2	21.2	21.2
D	25.0	25.0	25.0	25.3	25.0	25.3	25.1	25.3	25.1	25.2
E	16.7	14.3	14.1	13.3	13.5	13.1	13.3	13.1	13.3	13.2
F	8.3	7.1	6.3	6.0	5.7	5.8	5.6	5.7	5.6	5.7

fractions (1/12 = 0.83)

Eigenvector

- Mathematically, iterative degree centrality (iteratively summing the influences of nodes neighbours) converges on principal **eigenvector** of the adjacenty matrix.

- Eigenvector of a matrix is a non-zero vector that, when multiplied by the matrix, changes only in magnitude (not direction).

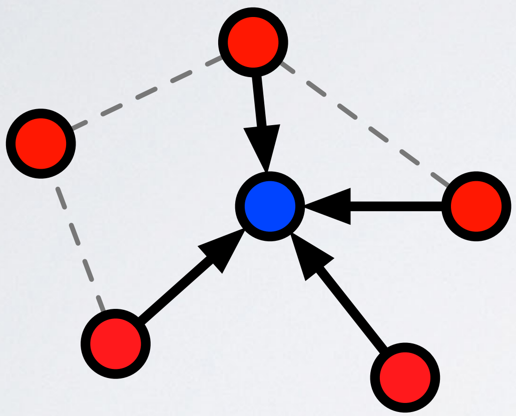
- The matrix A scales the vector v by the factor λ. The principal eigenvector corresponds to the largest eigenvalue of the matrix A.

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v} \qquad\qquad v_i = \frac{1}{\lambda}\sum_j a_{ij} v_j$$

v is the eigenvector, λ is the associated eigenvalue (a proportionality constant)

- A node has high eigenvector score to the extent it is connected to many nodes who themselves have high scores
- Often interpreted as popularity or status – have ties not just to many others but many <u>well-connected</u> others

eigenvector centrality



position = centrality:
PageRank, Katz, eigenvector centrality

importance = sum of importances* of nodes that point at you

$$I_i = \sum_{j \to i} \frac{I_j}{k_j}$$

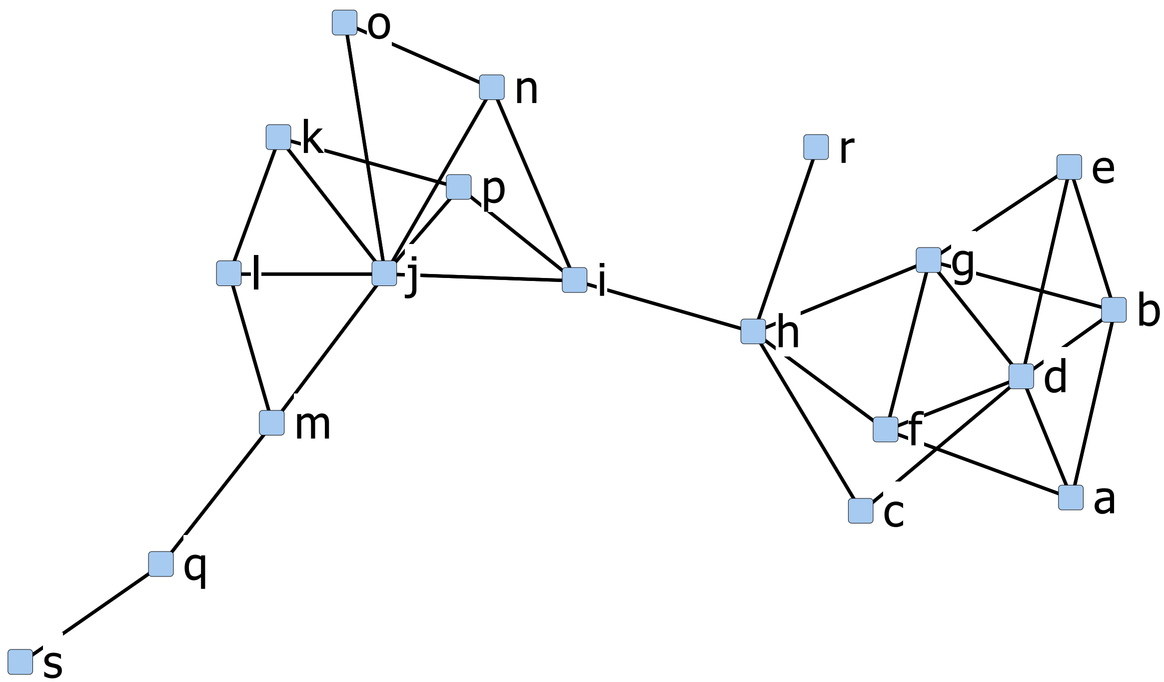
or, the left eigenvector of

$$\mathbf{A}\mathbf{x} = \lambda\,\mathbf{x}$$

borg4cent

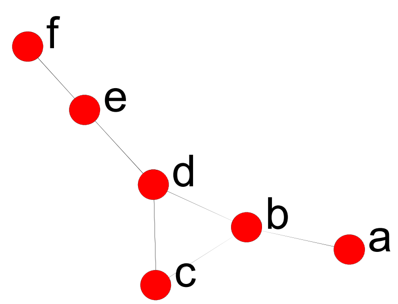
Eigenvector

- Node ***d*** has the highest eigenvector centrality in the land



Eigenvector can (usually) be computed as iterated degree*

	a	b	c	d	e	f	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
a	0	1	0	0	0	0	1	3	6	16	35	86	195	465	1071	2524
b	1	0	1	1	0	0	3	6	16	35	86	195	465	1071	2524	5854
c	0	1	0	1	0	0	2	6	13	32	73	173	401	940	2190	5117
d	0	1	1	0	1	0	3	7	16	38	87	206	475	1119	2593	6086
e	0	0	0	1	0	1	2	4	9	20	47	107	253	582	1372	3175
f	0	0	0	0	1	0	1	2	4	9	20	47	107	253	582	1372
							12	28	64	150	348	814	1896	4430	10332	24128



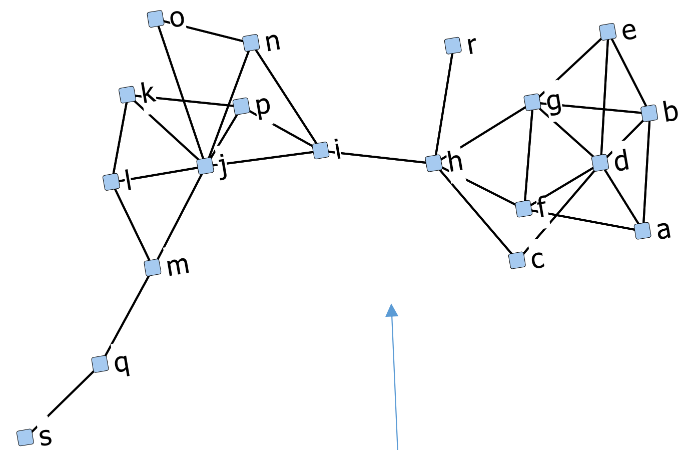
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
A	8.3	10.7	9.4	10.7	10.1	10.6	10.3	10.5	10.4	10.5
B	25.0	21.4	25.0	23.3	24.7	24.0	24.5	24.2	24.4	24.3
C	16.7	21.4	20.3	21.3	21.0	21.3	21.1	21.2	21.2	21.2
D	25.0	25.0	25.0	25.3	25.0	25.3	25.1	25.3	25.1	25.2
E	16.7	14.3	14.1	13.3	13.5	13.1	13.3	13.1	13.3	13.2
F	8.3	7.1	6.3	6.0	5.7	5.8	5.6	5.7	5.6	5.7

Node	D10	Eigen
a	10.5	0.234
b	24.3	0.545
c	21.2	0.475
d	25.2	0.564
e	13.2	0.296
f	5.7	0.127

r = 0.999992

*This is called the power method (Hotelling, 1930). Requires matrix to have unique dominant eigenvalue to converge.

Applications



- Playground status
  - You may have many friends, but if they are them-selves outcasts, it will not improve your status
  - You could have just one friend, but if this is the most popular kid in the school, your status will be good
- Being connected to those in the know – could be valuable
  - In principle, a better measure of exposure to what is flowing (cf *r* and *s*)
- Feeling well-grounded or anchored by circle of friends
- A measure of being in an in-crowd, a core, or dominant coalition

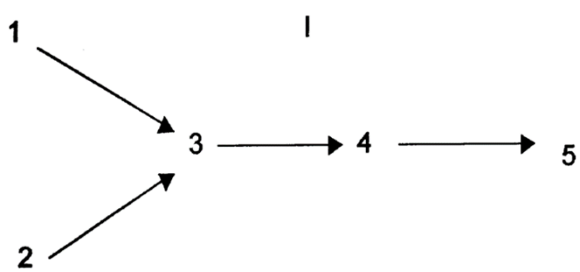
Issues with eigenvector

- Can’t use with disconnected networks
- In clumpy networks, it favors the nodes in the larger cliques
- It can fail as a measure of risk/exposure because it doesn’t take into account the fact that an alter’s high degree might be because of ties with nodes that ego is already connected to
    - So shouldn’t give that alter any weight, because they are not adding to exposure
- Many issues with directed data

Directed Eigenvector

$$l_j = \tfrac{1}{\lambda} \sum_i a_{ij} r_i$$

- In principle, similar to degree:
  - Out-eigenvector (known as right eigenvector) gives a high score to those who send to many people who themselves send to many people who …
    - If the relation is influences, then high score means you influence the influencers
  - In-eigenvector (left eigenvector) gives high score to those who receive from people who receive from many people who receive from …
    - For the respects relation, a high score indicates you are respected by the well respected
- In practice, is often not calculable or gives wacky answers



right

$$r_i = \frac{1}{\lambda} \sum_j a_{ij} r_j$$

left

$$l_j = \frac{1}{\lambda} \sum_i a_{ij} l_i$$

Beta centrality (aka Bonacich power, Bonacich 1987)

R1 = rowSums (degree)

$(I - \beta R)^{-1}$ rewritten as R^+ iff condition met.

- Defined as: $\boldsymbol{p} = (I - \beta R)^{-1} R1$
  - R is the adjacency matrix; $(I - \beta R)^{-1}$ is a new matrix derived from R
  - β is a parameter chosen by the user
- When -1/λ < β < 1/λ, where λ is largest eigenvalue of R, ***p*** can be seen as the row sums of this sum of matrices:

R⁺ is no. of walks, wtd inversely by length, btw each pair of nodes

R^+1 is a column vector giving the sum of each row of R^+

b^0R^1 equivalent to adjacency matrix.

b gets gradually smaller.

$$R^+ = b^0R^1 + b^1R^2 + b^2R^3 + b^3R^4 + \ldots$$

$$P = R^+1$$ (rowSums of R^+)

  - R2 gives the number of walks of exactly 2 steps between every pair of nodes
  - R3 gives the number of walks length 3 between all pairs of nodes, etc.
- Beta centrality measures # of walks of all lengths, weighted inversely by length, that emanate from a node

if ß = 0.5, $b^0 = 1$, $b^1 = 0.5$, $b^2 = 0.25$, $b^3 = 0.13$, $b^4 = 0.06$, etc

The β parameter in beta centrality

$$R^{+} = b^{0}R^{1} + b^{1}R^{2} + b^{2}R^{3} + b^{3}R^{4} + \ldots$$

$$P = R^{+}1$$

- When β is 0, beta centrality equals degree
  - Only paths of length 1 (direct connections) matter
- As β increases from 0, longer paths are given increasing weight
- When β is as close to 1/λ as possible, beta centrality equals eigenvector centrality
- When β gets larger than 1/λ, beta centrality becomes uninterpretable

Degree ⟷ Eigenvector

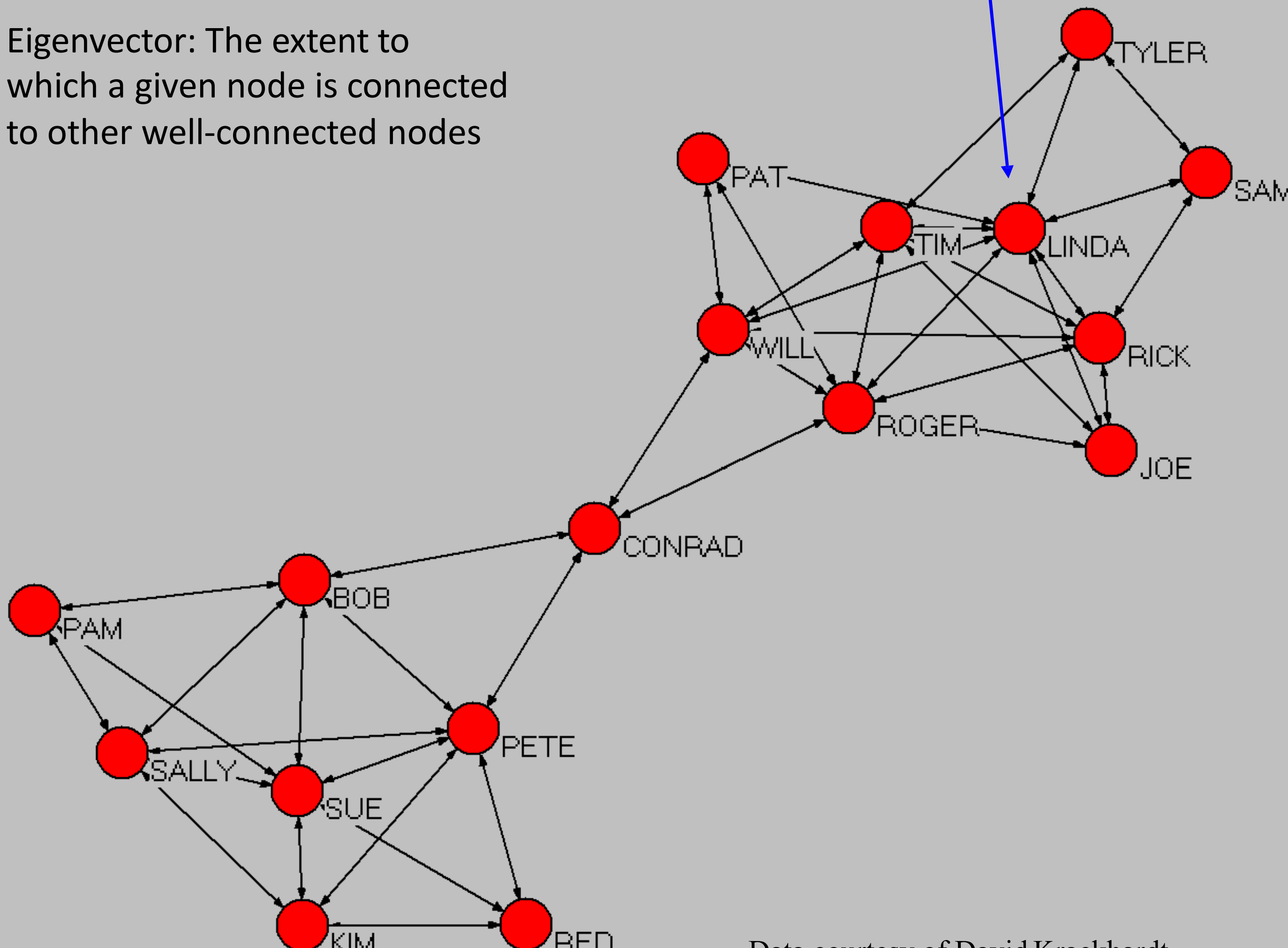
$\beta = 0$ | β *centrality* | $\beta \approx 1/\lambda$

Issues with beta centrality

- Often highly related to degree
- How to choose beta?
- But … it works great with directed graphs

Eigenvector

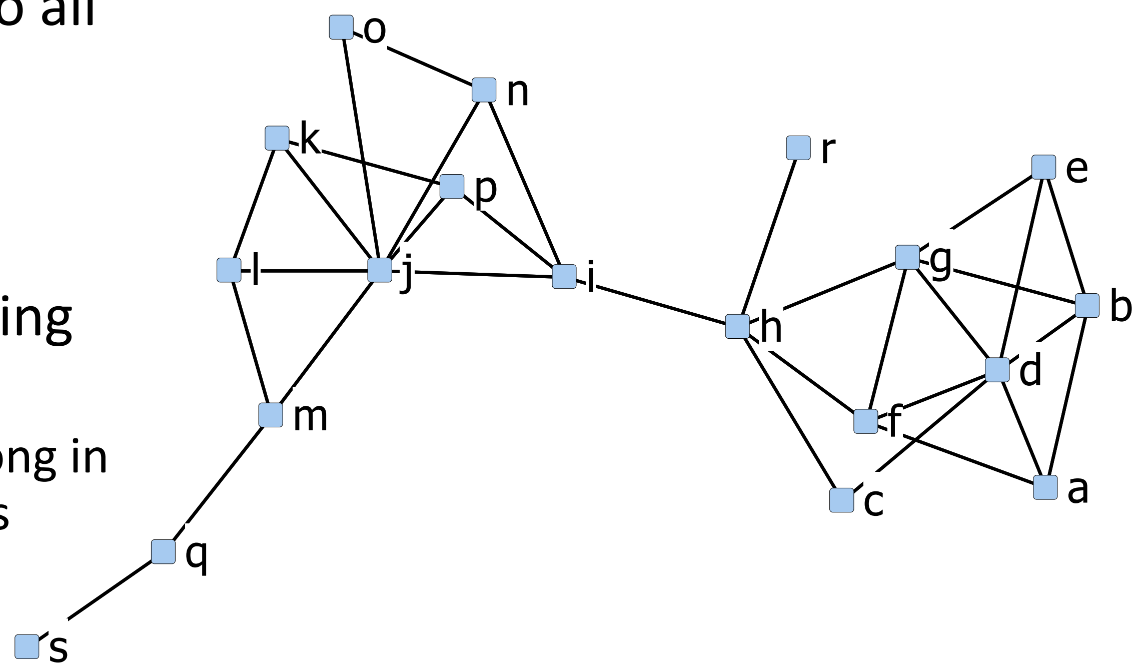
Eigenvector: The extent to which a given node is connected to other well-connected nodes



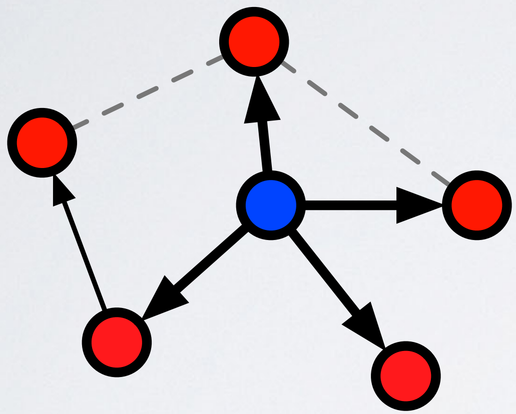
Data courtesy of David Krackhardt

Closeness

- Sum of distances from node to all others
- Inverse measure of centrality
- Often interpreted as index of time-until-arrival of stuff flowing through network
  - In gossip network, persons strong in closeness centrality hear things early



describing networks



position = centrality:
harmonic, closeness centrality

importance = being in "center" of the network

harmonic $$c_i = \frac{1}{n-1}\sum_{j\neq i}\frac{1}{d_{ij}}$$

length of shortest path

distance: $$d_{ij} = \begin{cases} \ell_{ij} & \text{if } j \text{ reachable from } i \\ \infty & \text{otherwise} \end{cases}$$

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, *Social Networks* **27**, 55–71 (2005)

closeness centrality formula

$$\tilde{C}^C(i) = \left[\sum_{j=1}^{N} d(i,j)\right]^{-1}$$

Normalized

$$C^C(i) = \frac{\tilde{C}^C(i)}{N-1}$$

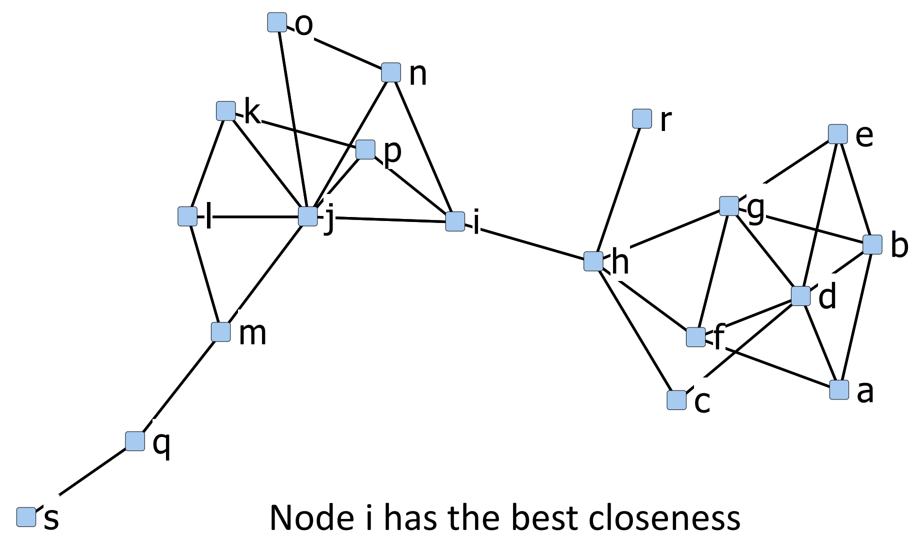
All other nodes in the network

What happens to isolates?

Closeness as marginals of distance matrix

ID	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	sum
a	0	1	2	1	2	1	2	2	3	4	5	5	5	4	5	4	6	3	7	62
b	1	0	2	1	1	2	1	2	3	4	5	5	5	4	5	4	6	3	7	61
c	2	2	0	1	2	2	2	1	2	3	4	4	4	3	4	3	5	2	6	52
d	1	1	1	0	1	1	1	2	3	4	5	5	5	4	5	4	6	3	7	59
e	2	1	2	1	0	2	1	2	3	4	5	5	5	4	5	4	6	3	7	62
f	1	2	2	1	2	0	1	1	2	3	4	4	4	3	4	3	5	2	6	50
g	2	1	2	1	1	1	0	1	2	3	4	4	4	3	4	3	5	2	6	49
h	2	2	1	2	2	1	1	0	1	2	3	3	3	2	3	2	4	1	5	40
i	3	3	2	3	3	2	2	1	0	1	2	2	2	1	2	1	3	2	4	39
j	4	4	3	4	4	3	3	2	1	0	1	1	1	1	1	1	2	3	3	42
k	5	5	4	5	5	4	4	3	2	1	0	1	2	2	2	1	3	4	4	57
l	5	5	4	5	5	4	4	3	2	1	1	0	1	2	2	2	2	4	3	55
m	5	5	4	5	5	4	4	3	2	1	2	1	0	2	2	2	1	4	2	54
n	4	4	3	4	4	3	3	2	1	1	2	2	2	0	1	2	3	3	4	48
o	5	5	4	5	5	4	4	3	2	1	2	2	2	1	0	2	3	4	4	58
p	4	4	3	4	4	3	3	2	1	1	1	2	2	2	2	0	3	3	4	48
q	6	6	5	6	6	5	5	4	3	2	3	2	1	3	3	3	0	5	1	69
r	3	3	2	3	3	2	2	1	2	3	4	4	4	3	4	3	5	0	6	57
s	7	7	6	7	7	6	6	5	4	3	4	3	2	4	4	4	1	6	0	86
sum	62	61	52	59	62	50	49	40	39	42	57	55	54	48	58	48	69	57	86	1048

Geodesic distances matrix (smallest numbers are MOST central, best closeness).



Node i has the best closeness

Average distance would be more interpretable

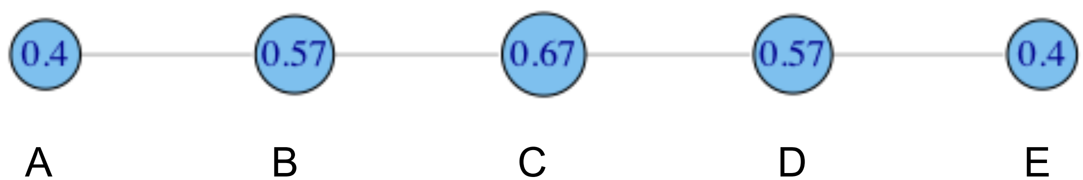
Reciprocal Distance

	a	b	c	d	e	f	g	h	i	j
a	0	1	1	1	2	3	4	5	4	5
b	1	0	1	2	1	2	3	4	3	4
c	1	1	0	1	2	3	4	5	4	5
d	1	2	1	0	1	2	3	4	3	4
e	2	1	2	1	0	1	2	3	2	3
f	3	2	3	2	1	0	1	2	1	2
g	4	3	4	3	2	1	0	1	2	1
h	5	4	5	4	3	2	1	0	1	1
i	4	3	4	3	2	1	2	1	0	1
j	5	4	5	4	3	2	1	1	1	0

	a	b	c	d	e	f	g	h	i	j
a	0.00	1.00	1.00	1.00	0.50	0.33	0.25	0.20	0.25	0.20
b	1.00	0.00	1.00	0.50	1.00	0.50	0.33	0.25	0.33	0.25
c	1.00	1.00	0.00	1.00	0.50	0.33	0.25	0.20	0.25	0.20
d	1.00	0.50	1.00	0.00	1.00	0.50	0.33	0.25	0.33	0.25
e	0.50	1.00	0.50	1.00	0.00	1.00	0.50	0.33	0.50	0.33
f	0.33	0.50	0.33	0.50	1.00	0.00	1.00	0.50	1.00	0.50
g	0.25	0.33	0.25	0.33	0.50	1.00	0.00	1.00	0.50	1.00
h	0.20	0.25	0.20	0.25	0.33	0.50	1.00	0.00	1.00	1.00
i	0.25	0.33	0.25	0.33	0.50	1.00	0.50	1.00	0.00	1.00
j	0.20	0.25	0.20	0.25	0.33	0.50	1.00	1.00	1.00	0.00

For undefined distances, we can define the reciprocal distance to be 0

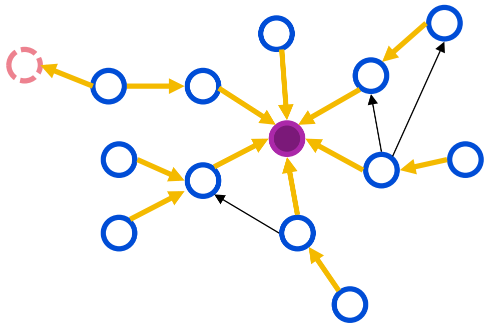
closeness centrality example



$$C'_c(A) = \left[\frac{\sum_{j=1}^{N} d(A,j)}{N-1} \right]^{-1} = \left[\frac{1+2+3+4}{4} \right]^{-1} = \left[\frac{10}{4} \right]^{-1} = 0.4$$

Closeness in directed networks

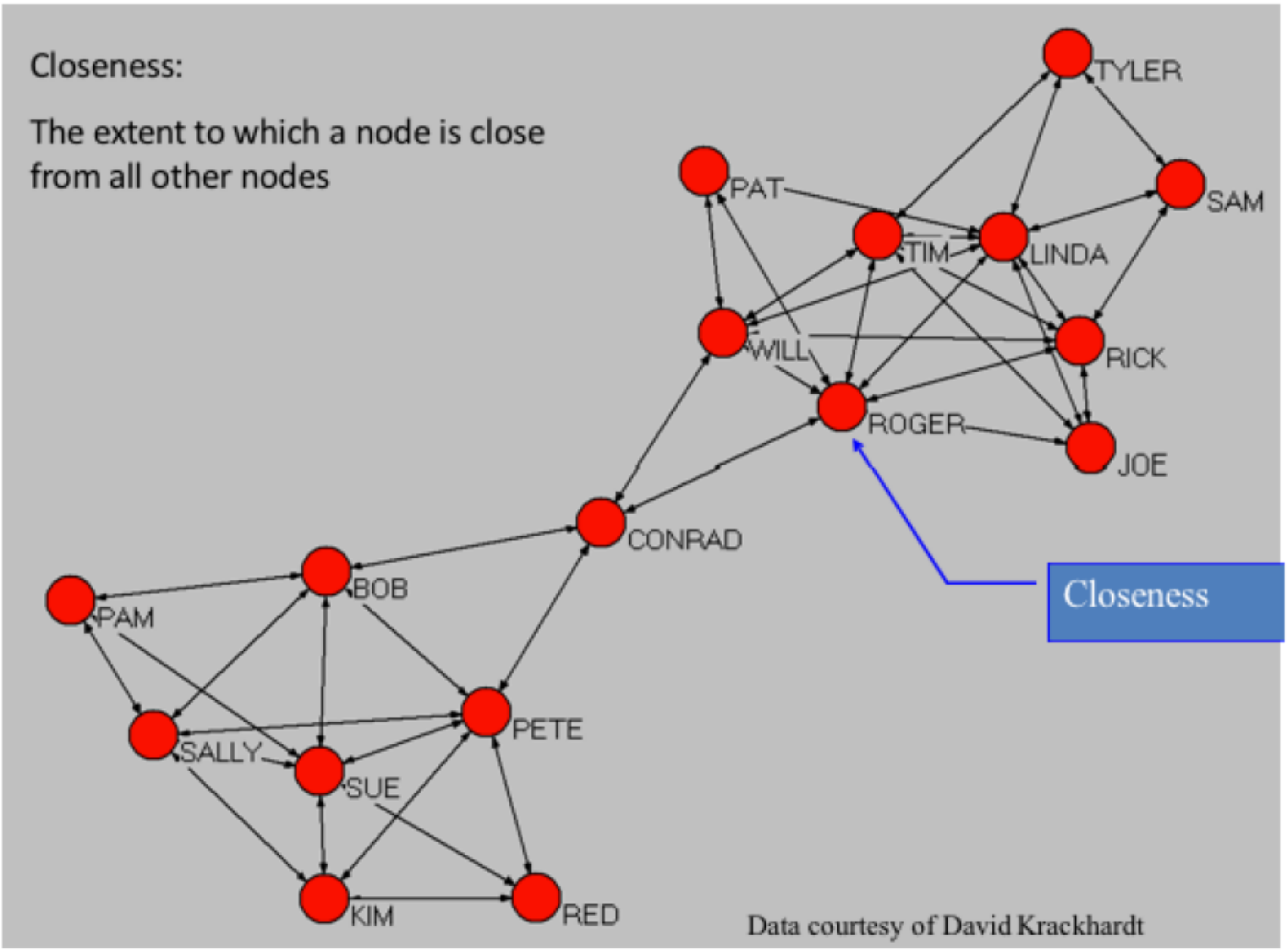
- choose a direction
  - in-closeness (e.g. prestige in citation networks)
  - out-closeness
- usually consider only vertices from which the node *i* in question can be reached



Applications

- Any situation where the value of information (or the cost of infection) is a function of time
  - Getting a disease before there is any treatment available
  - Getting gossip before most people have already heard it
  - Getting market information before other investors have heard it
- Nodes with the best closeness scores are often the ones to tap to learn what the network knows
  - Picking a snitch/key informant/potential spy

Closeness


Closeness:
The extent to which a node is close from all other nodes
TYLER
PAT
SAM
TIM
LINDA
WILL
RICK
ROGER
JOE
CONRAD
BOB
PAM
PETE
SALLY
SUE
KIM
RED
Closeness
Data courtesy of David Krackhardt

Issues & variants of closeness

- Only looks at shortest paths
- When graphs are disconnected, distances between some nodes are undefined
  - What to do?
- One approach is average reciprocal distance (ARD)
  - Replace entries dij of distance matrix with 1/dij, and set 1/dij to zero when dij is undefined
  - Take the average across all other nodes $ARD(i) = j \sum \frac{1}{d_{ij}}$
- Another approach is k-reach: the proportion of other a node can reach within k steps

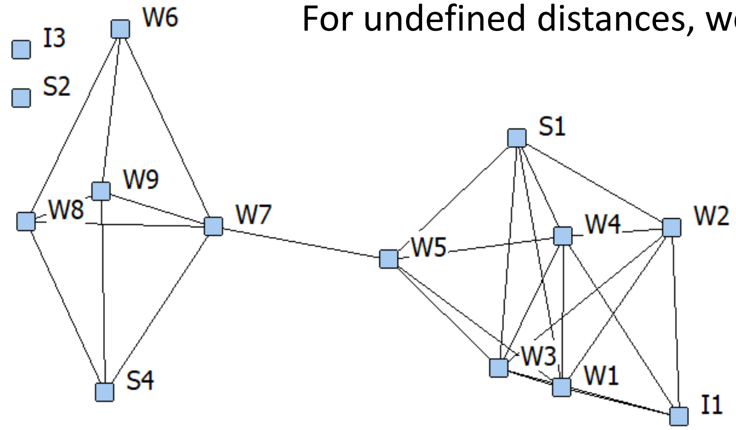
Geodesic distance matrix

- How to get row or col sums when you have undefined distances?

Do the reciprocal (1/infinity=0) or add large value.

	HOLLY	BRAZEY	CAROL	PAM	PAT	JENNIE	PAULINE	ANN	MICHAEL	BILL	LEE	DON	JOHN	HARRY	GERY	STEVE	BERT	RUSS
HOLLY	0		2	1	1	2	2	2	2			1		2				
BRAZEY	5	0	7	6	6	7	7	7	4		1	5		5	3	1	1	2
CAROL	2		0	1	1	2	1	2	4			3		4				
PAM	3		2	0	2	1	1	1	5			4		5				
PAT	1		1	2	0	1	2	2	3			2		3				
JENNIE	2		2	1	1	0	2	1	4			3		4				
PAULINE	2		1	1	1	2	0	2	4			3		4				
ANN	3		2	1	2	1	1	0	5			4		5				
MICHAEL	1		3	2	2	3	3	3	0			1		1				
BILL	2		4	3	3	4	4	4	1	0		1		1				
LEE	5	1	7	6	6	7	7	7	4		0	5		5	3	1	1	2
DON	1		3	2	2	3	3	3	1			0		1				
JOHN	3	4	2	2	2	3	1	3	2		3	3	0	3	1	2	2	1
HARRY	1		3	2	2	3	3	3	1			1		0				
GERY	2	3	4	3	3	4	4	4	1		2	2		2	0	1	2	1
STEVE	4	2	6	5	5	6	6	6	3		1	4		4	2	0	1	1
BERT	4	2	6	5	5	6	6	6	3		1	4		4	2	1	0	1
RUSS	3	3	5	4	4	5	5	5	2		2	3		3	1	1	1	0

For undefined distances, we can define the reciprocal distance to be 0



	I1	I3	W1	W2	W3	W4	W5	W6	W7	W8	W9	S1	S2	S4
I1	0		1	1	1	1	2	4	3	4	4	2		4
I3		0												
W1	1		0	1	1	1	1	3	2	3	3	1		3
W2	1		1	0	1	1	2	4	3	4	4	1		4
W3	1		1	1	0	1	1	3	2	3	3	1		3
W4	1		1	1	1	0	1	3	2	3	3	1		3
W5	2		1	2	1	1	0	2	1	2	2	1		2
W6	4		3	4	3	3	2	0	1	1	1	3		2
W7	3		2	3	2	2	1	1	0	1	1	2		1
W8	4		3	4	3	3	2	1	1	0	1	3		1
W9	4		3	4	3	3	2	1	1	1	0	3		1
S1	2		1	1	1	1	1	3	2	3	3	0		3
S2													0	
S4	4		3	4	3	3	2	2	1	1	1	3		0

	I1	I3	W1	W2	W3	W4	W5	W6	W7	W8	W9	S1	S2	S4
I1		0	1	1	1	1	0.5	0.25	0.33	0.25	0.25	0.5	0	0.25
I3	0		0	0	0	0	0	0	0	0	0	0	0	0
W1	1	0		1	1	1	1	0.33	0.5	0.33	0.33	1	0	0.33
W2	1	0	1		1	1	0.5	0.25	0.33	0.25	0.25	1	0	0.25
W3	1	0	1	1		1	1	0.33	0.5	0.33	0.33	1	0	0.33
W4	1	0	1	1	1		1	0.33	0.5	0.33	0.33	1	0	0.33
W5	0.5	0	1	0.5	1	1		0.5	1	0.5	0.5	1	0	0.5
W6	0.25	0	0.33	0.25	0.33	0.33	0.5		1	1	1	0.33	0	0.5
W7	0.33	0	0.5	0.33	0.5	0.5	1	1		1	1	0.5	0	1
W8	0.25	0	0.33	0.25	0.33	0.33	0.5	1	1		1	0.33	0	1
W9	0.25	0	0.33	0.25	0.33	0.33	0.5	1	1	1		0.33	0	1
S1	0.5	0	1	1	1	1	1	0.33	0.5	0.33	0.33		0	0.33
S2	0	0	0	0	0	0	0	0	0	0	0	0		0
S4	0.25	0	0.33	0.25	0.33	0.33	0.5	0.5	1	1	1	0.33	0	

K-Reach centrality

- Proportion of others that ego can reach by a path of k or less
  - 1-reach is just normalized degree centrality
- Highly interpretable. Holly can reach 65% of the network in 2 steps, so she is a good influencer

```
->tcamp = symmet(campnet)
Network|Centrality|Reach ~tcamp
```

Node-by-Distance Proportion of Nodes Reached Matrix

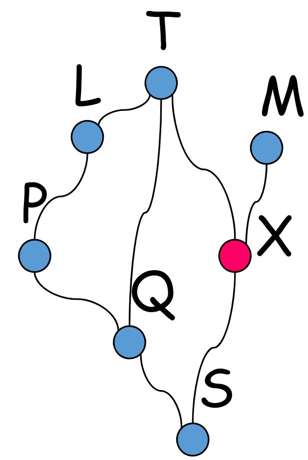
		1 d1	2 d2	3 d3	4 d4	5 d5	6 d6
1	HOLLY	0.29	0.65	0.82	1.00	1.00	1.00
2	BRAZEY	0.18	0.29	0.41	0.71	0.94	1.00
3	CAROL	0.18	0.41	0.71	0.88	1.00	1.00
4	PAM	0.29	0.59	0.76	0.88	1.00	1.00
5	PAT	0.24	0.59	0.76	0.88	1.00	1.00
6	JENNIE	0.18	0.35	0.59	0.76	0.88	1.00
7	PAULINE	0.29	0.53	0.82	1.00	1.00	1.00
8	ANN	0.18	0.41	0.71	0.88	1.00	1.00
9	MICHAEL	0.29	0.59	1.00	1.00	1.00	1.00
10	BILL	0.18	0.29	0.59	1.00	1.00	1.00
11	LEE	0.18	0.29	0.41	0.71	0.94	1.00
12	DON	0.24	0.41	0.82	1.00	1.00	1.00
13	JOHN	0.18	0.59	1.00	1.00	1.00	1.00
14	HARRY	0.24	0.41	0.82	1.00	1.00	1.00
15	GERY	0.24	0.71	0.94	1.00	1.00	1.00
16	STEVE	0.29	0.41	0.71	0.94	1.00	1.00
17	BERT	0.24	0.35	0.47	0.94	1.00	1.00
18	RUSS	0.24	0.47	0.94	1.00	1.00	1.00

Betweenness

- Loosely, the extent to which a node is along the shortest paths of between all pairs of nodes

$$b_k = \sum_{i,j} \frac{g_{ikj}}{g_{ij}}$$

gij is number of geodesic paths from i to j
gikj is number of geodesics from i to j that pass through k



- More correctly, bk is the share of geodesics between pairs of nodes that pass through k
- Often interpreted as control over flows (gatekeeping), correlated with power
- Also seen as index of frequency something reaches node

formula

$$\tilde{C}^B(i) = \sum_{j<k} \frac{d_{jk}(i)}{d_{jk}}$$

d_{jk} # of shortest paths between j and k

$d_{jk}(i)$ # of shortest paths between j and k that go through i

Normalized

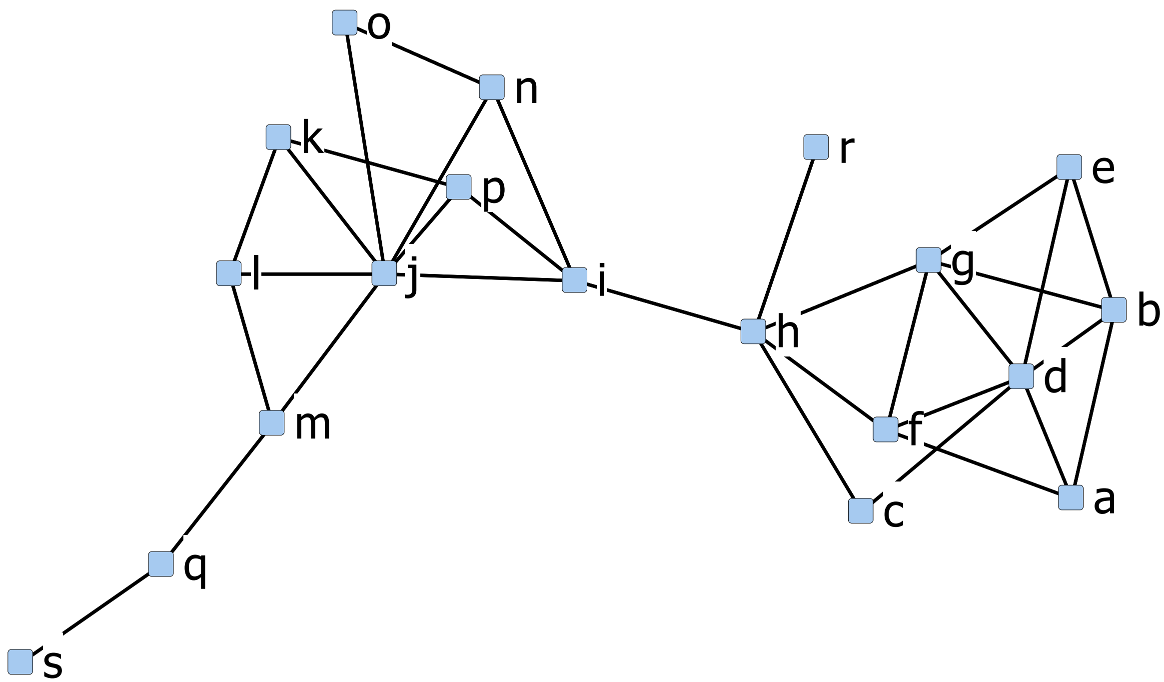
$$C^B(i) = \frac{\tilde{C}^B}{(N-1)(N-2)/2}$$

Number of pairs of vertices excluding i

For **directed graphs**: when normalizing, we have (N-1)*(N-2) instead of (N-1)*(N-2)/2, because we have twice as many ordered pairs as unordered pairs.

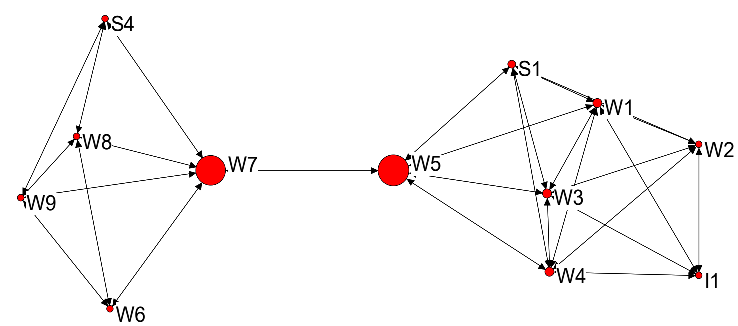
Betweenness

- Node *h* has the highest betweenness



Betweenness – cont.

- Often discussed in terms of identifying liaisons, gatekeepers, "secretary power"
- Global network cohesion is highly dependent on high betweenness nodes.
  - (But) networks that contain high betweenness nodes are brittle
- Nodes with high betweenness and low degree are often overlooked by network members themselves
  - Degree is highly visible, betweenness may not be

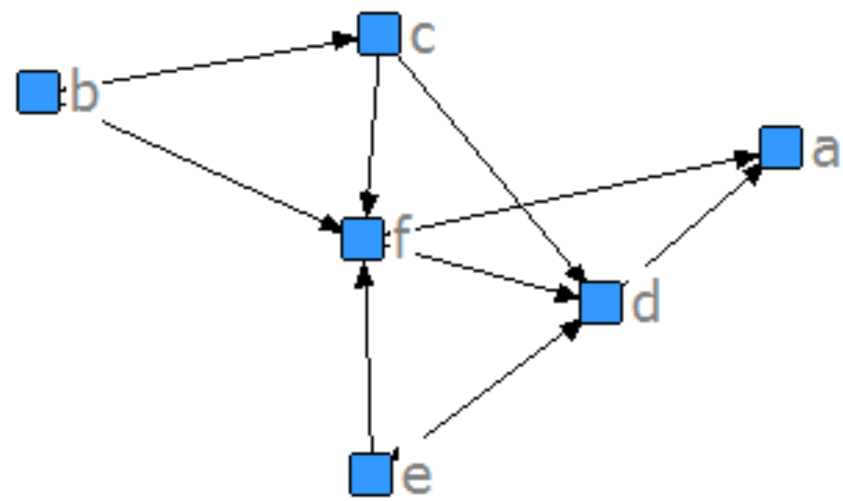


Betweenness

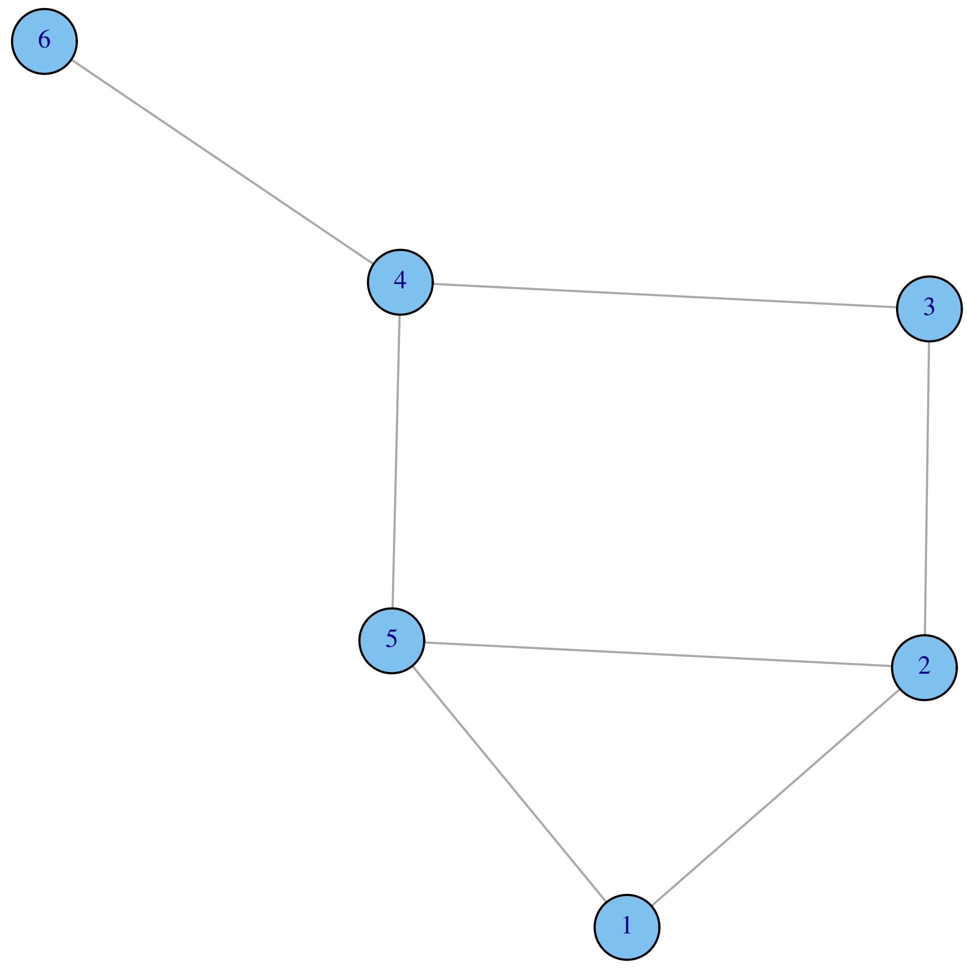
- With betweenness, there is no need for separate in and out versions
  - A node is between two others if it is along a directed path from one to the other

F gets no points for being between E and B, because there is no directed path from E to B

- B has only outgoing arrows, so no way to get <u>to</u> B



Betweenness in directed networks



Betweenness in directed networks

- For example: for node 2, the $(n-1)(n-2)/2 = 5(5-1)/2 = 10$ terms in the summation in the order of 13, 14, 15, 16, 34, 35, 36, 45, 46, 56 are

$$\frac{1}{1} + \frac{0}{1} + \frac{0}{1} + \frac{0}{1} + \frac{0}{1} + \frac{1}{2} + \frac{0}{1} + \frac{0}{1} + \frac{0}{1} + \frac{0}{1} = 1.5.$$

- Here the denominators are the number of shortest paths between pair of edges in the above order and the numerators are the number of shortest paths passing through edge 2 between pair of edges in the above order.

Applications

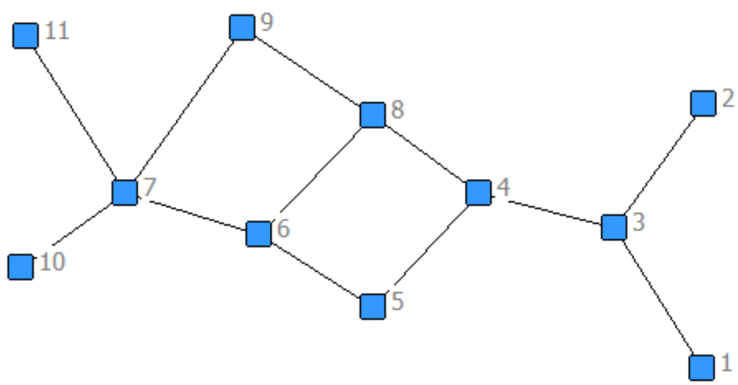
- Often associated with power (Brass, 1984)
- High betweenness nodes (if they know they are in that position) can extract rents for passing things along or introducing people
- Betweenness works best with hard-to-form ties, like roads or trust ties
  - Otherwise nodes can bypass the high betweenness node by connecting directly with others
- At a crossroads in the network. Paths may not be short, but flows are fairly certain to pass through the node

e.g., Mehra, A., Kilduff, M. and Brass, D.J., 2001. The social networks of high and low self-monitors: Implications for workplace performance. *Administrative science quarterly*, *46*(1), pp.121-146.

Duality of closeness & betweenness

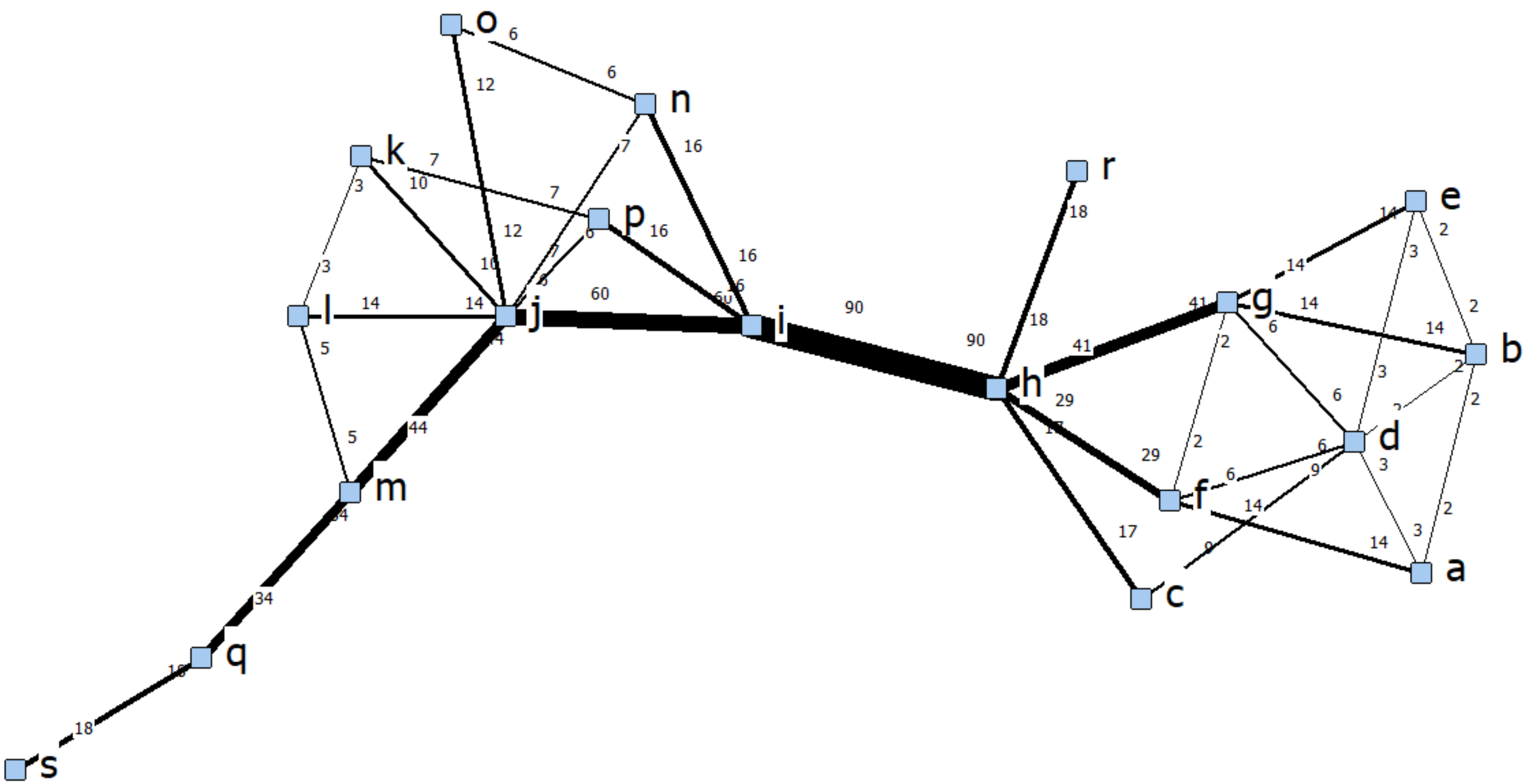
	Closen	Betwee
	------	------
1	36.000	0.000
2	36.000	0.000
3	27.000	17.000
4	22.000	21.833
5	23.000	6.000
6	22.000	13.667
7	25.000	17.833
8	21.000	15.167
9	24.000	5.500
10	34.000	0.000
11	34.000	0.000

- Dependency matrix D, where dij = number of times* that i needs to go through j to reach someone via a shortest path
- Column totals of D equal betweenness times 2
- Row totals of D equal closeness minus n-1

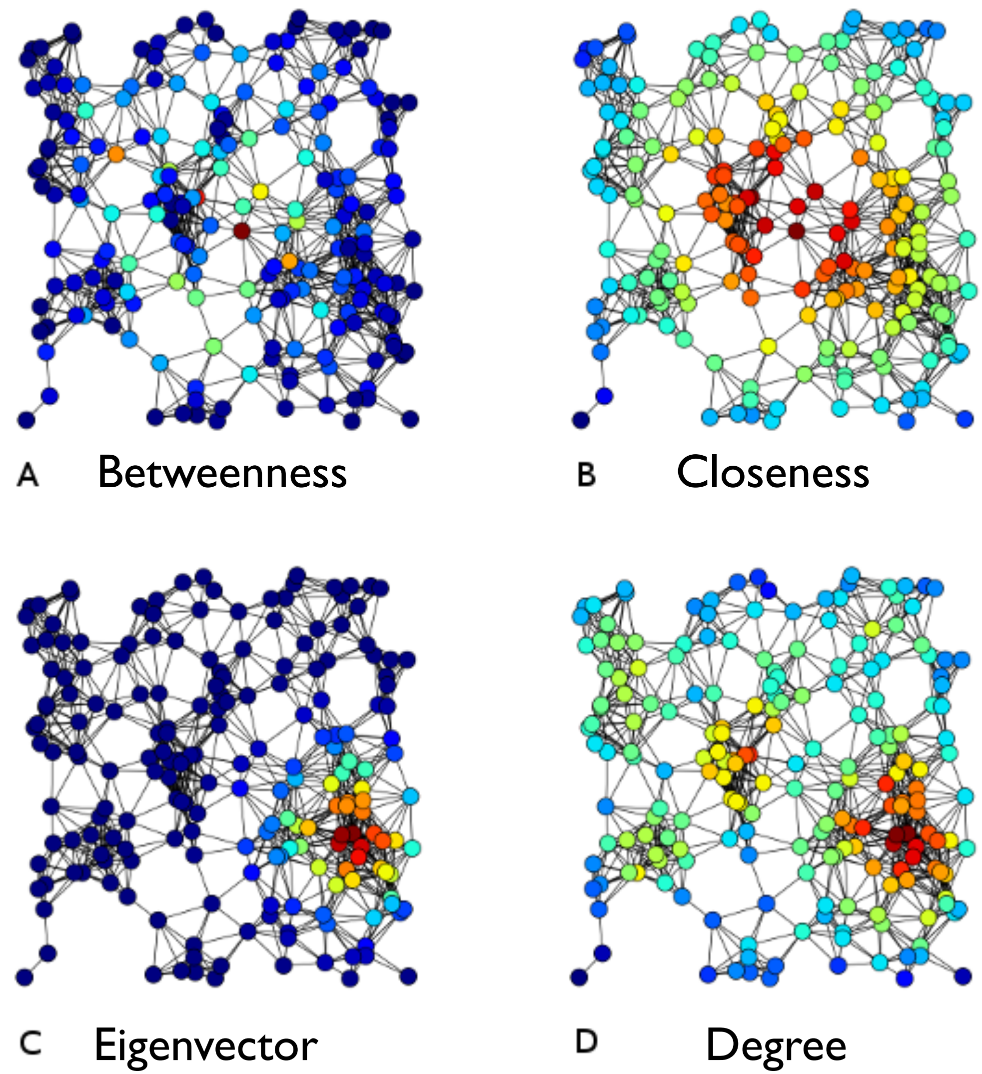


	1	2	3	4	5	6	7	8	9	10	11	Clo
1		0.00	9.00	7.00	1.50	2.00	2.00	3.50	1.00	0.00	0.00	26.00
2	0.00		9.00	7.00	1.50	2.00	2.00	3.50	1.00	0.00	0.00	26.00
3	0.00	0.00		7.00	1.50	2.00	2.00	3.50	1.00	0.00	0.00	17.00
4	0.00	0.00	2.00		1.50	2.00	2.00	3.50	1.00	0.00	0.00	12.00
5	0.00	0.00	2.00	3.83		4.17	2.33	0.67	0.00	0.00	0.00	13.00
6	0.00	0.00	2.00	3.00	2.00		2.50	2.50	0.00	0.00	0.00	12.00
7	0.00	0.00	2.00	3.00	1.33	4.17		2.67	1.83	0.00	0.00	15.00
8	0.00	0.00	2.00	3.50	0.00	2.00	2.00		1.50	0.00	0.00	11.00
9	0.00	0.00	2.00	3.33	0.00	0.67	2.83	5.17		0.00	0.00	14.00
10	0.00	0.00	2.00	3.00	1.33	4.17	9.00	2.67	1.83		0.00	24.00
11	0.00	0.00	2.00	3.00	1.33	4.17	9.00	2.67	1.83	0.00		24.00
Bet	0.00	0.00	34.00	43.67	12.00	27.33	35.67	30.33	11.00	0.00	0.00	194.00

Betweenness of edges



Centrality indices are answers to the question "**<u>What characterizes an important node?</u>**"
The word "importance" has a <u>wide number of meanings</u>, leading to <u>many different definitions of centrality</u>.



Measures and type of network

Graph	Degree	Eigenvector	Beta Centrality	Closeness	Betweenness
Undirected	Ok	Ok	Ok	Ok	Ok
Directed	Ok	Very problematic	Ok	Problematic[a]	Ok
Valued	Ok	Ok	Ok	No[b]	No[c]
Disconnected	Ok	No	No	No	Ok

[a] only a problem because directed graphs are often disconnected -- have unreachable nodes
[b] there are ways to do it in ucinet, but not commonly accepted
[c] not possible in Ucinet, but in principle can be done easily with values that represent costs or distances

check your understanding

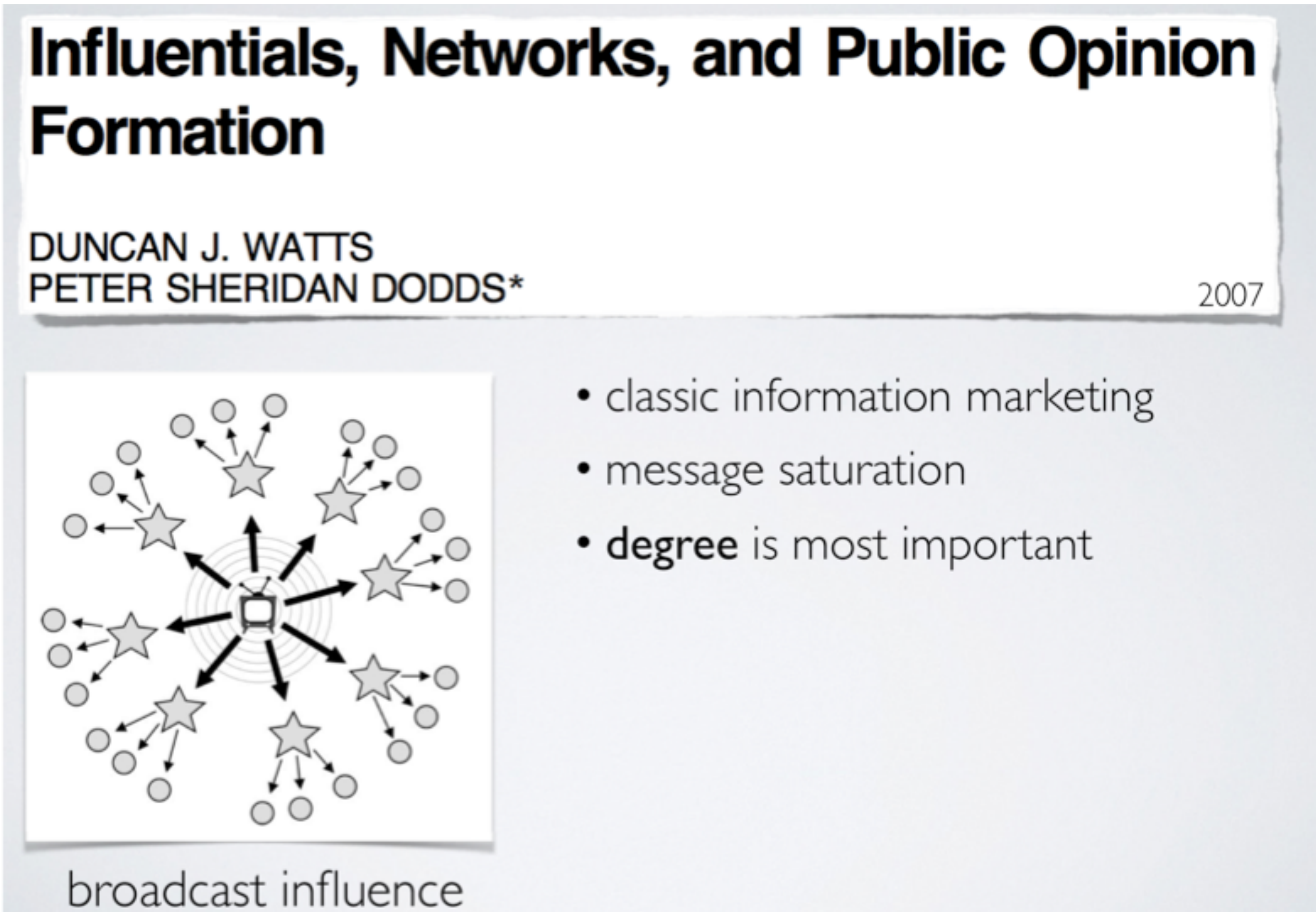
- generally different centrality metrics will be positively correlated
- when they are not, there is likely something interesting about the network
- suggest possible topologies and node positions to fit each square

	Low Degree	Low Closeness	Low Betweenness
High Degree			
High Closeness			
High Betweenness			

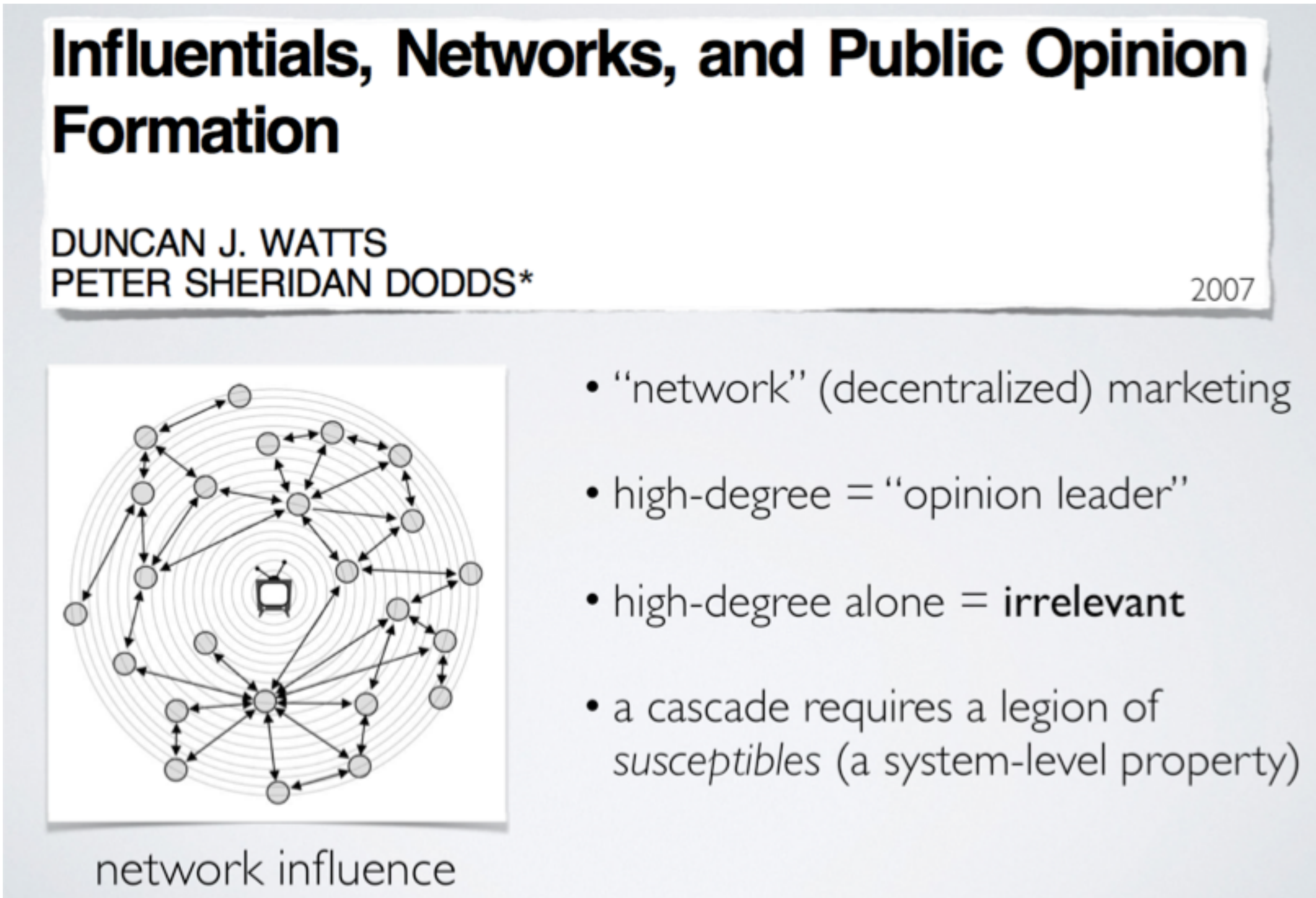
- generally different centrality metrics will be positively correlated
- when they are not, there is likely something interesting about the network
- suggest possible topologies and node positions to fit each square

	Low Degree	Low Closeness	Low Betweenness
High Degree		Embedded in cluster that is far from the rest of the network	Ego's connections are redundant - communication bypasses him/her
High Closeness	Key player tied to important/active players		Probably multiple paths in the network, ego is near many people, but so are many others
High Betweenness	Ego's few ties are crucial for network flow	Very rare cell. Would mean that ego monopolizes the ties from a small number of people to many others.	

“model in which opinion flows only from the media to influentials, and then only from influentials to the larger populace is deprecated”



“large cascades of influence are driven not by influentials, but by a critical mass of easily influenced individuals.”



“influence is not really about the influencer as much about the susceptibles”